Caustics, catastrophes and quantum chaos

THEORETICIAN, PROFESSOR SIR MICHAEL BERRY FRS, RECEIVED HIS KNIGHTHOOD LAST YEAR FOR SERVICES TO PHYSICS. HE TELLS NINA HALL HOW HE MAKES THE CONNECTIONS BETWEEN PHYSICS AND MATHEMATICS . . .

"Has my colour printer arrived yet?," Michael Berry is anxiously hopping up and down in front of his secretary Tracie, as I step through the door of his office in the physics department at Bristol University. I have to come to interview one of the most congenial as well as the most respected researchers in British physics. He greets me warmly; we haven't met up for about eight years, and want to catch up on news. But the newly ordered colour printer has not appeared and Sir Michael is annoyed, having got up before dawn to await an early delivery.

I was curious to know why the colour printer was so important. Berry explained with characteristic enthusiasm that it will help him with his latest interest, coloured caustics - the delicately patterned chromatic distortions that form when white light is partially focused and diffracted through an irregular medium such as raindrops on a glass surface or a frosted bathroom window. "I had noticed these caustics with my naked eye - intermittently, over many years - and was puzzled by some of the fine interference fringes within them," says Berry. One of the effects that intrigued him was that fringes produced by white light were black and white instead of being split into colours. He later realised that this was because the fringes were too fine to be resolved by the colour-sensitive cones of the eye.

'I say I'm studying the physics of the mathematics of the physics'

Physics in its true colours

Berry has been investigating the caustics using mathematics to simulate their shapes, with the aid of an Apple Mac and the program Mathematica. "I've had to learn a lot of different technologies to render the coloured patterns accurately." He explains that it's hard to use colour to give a true representation of wavelengths and intensities: "First you have to know about how the eye sees colour, and then you have to calibrate the screen in terms of voltages applied to the red, green and blue elements of each pixel. That demands knowing the characteristic of the phosphors, which I had to measure. The screen is also nonlinear so I've had to measure that too. It's really fun, and a wonderful example of science and technology coming together."

As a physicist in his 50s, Sir Michael Berry still exudes this infectious enthusiasm in explaining all his ideas in physics. Yet he eschews the outward conceits of scientific success, and wears his knighthood lightly. Berry received it for work that spans mathematics, optics and quantum mechanics (a particular quantum property is named after him - Berry's phase). Much of the work has implications in other areas such as condensed matter physics, chemistry and number theory.

Berry's research has often revealed in subtle ways the deep relationship between physics and mathematics, of which he is very aware. His approach has been to develop the mathematical techniques to deal with a general physical problem and then look for a physical analogue to represent the mathematical structure. The coloured caustics of raindrops on glass offer just such a model. "I say I'm studying the physics of the mathematics of the physics," says Berry. "For me, mathematical structures only come alive when I think of them in a geometrical, physical way."

'The aim is to sew the quantum flesh on classical bones'

Yet Berry admits it wasn't until he was in his 30s that he established this style of working which was to make his name. Surprisingly, he had not found theory easy when he was doing his first degree at Exeter - a comforting thought for any undergraduate struggling for the first time through wave theory. "I never studied mathematics properly in my life. The undergraduate degree was largely experimental. Then when I went to St Andrews to do a PhD in theory I was really floundering. I had to learn it all by myself which I didn't find particularly easy, although I was excited by the ideas."

However, under the tutelage of his PhD supervisor Bob Dingle, Berry developed a love for the branch of mathematics that has been the driving force behind most of his research ever since. This was the subject of asymptotics which was greatly advanced by Dingle and concerns divergent series - an area whose significance in physics is only now starting to be recognised. Berry's PhD involved applying asymptotics to the theoretically awkward problem of diffraction of light by ultrasound.

Dealing with infinity

From this initial work Berry developed an approach he calls physical asymptotics, which he jokingly says is humanity's latest way of dealing with infinity, "The best mathematical work I've done has been to develop some technologies for dealing with divergent series." Berry points out their importance: "If you can't a solve a problem in physics exactly then you try to represent it mathematically as an infinite series, hoping that it will converge. In fact almost all series describing physics diverge."

A prime example where divergent series are significant is in semi-classical physics - the border country between quantum mechanics and classical mechanics where Berry was to establish himself when he moved in 1965 to Bristol after his PhD. "I realised that semi-classical physics was the natural setting for these sophisticated asymptotic ideas", says Berry.

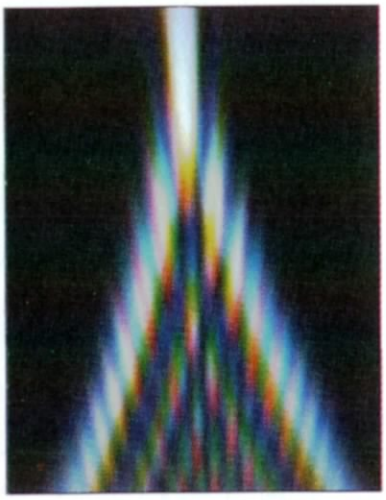
Semi-classical quantum mechanics was not then a fashionable subject, but in the 1970s, two mathematical developments came along which had a huge impact on Berry's work. The first was catastrophe theory, developed by René Thom, which described how a smooth variation in a system could trigger an abrupt change in response. It helped Berry to solve a particular problem in semi-classical theory, where the approach used is to take the equation for a classical system (for example, electrons scattered by a crystal atomic lattice) and then dress it in the wave theory of quantum mechanics. "The aim is to sew the quantum flesh on classical bones," says Berry. But the theory fails when the electron waves get focused into regions of high intensity - the quantum equivalent of the caustics seen in classical optical systems.

"Catastrophe theory provided a library of shapes that the caustics could take, and I was able to use it to solve the general problem of how to tailor the quantum flesh under these near-focusing conditions. I realised I could do many things with catastrophe theory especially in the field of optics," says Berry.

Music of the primes

The second influential area was chaos theory. Chaos appears in quite simple classical systems such as a pendulum driven by motor swinging over a magnet. The pendulum repeatedly explores the same space but never follows exactly the same trajectory. Berry was interested in how far such chaotic behaviour can penetrate into the quantum regime. The simplest model for study is an electron in a hydrogen atom moving in a strong magnetic field in an orbit far from the nucleus. Here, the electron's behaviour wavers between quantum and classical. Experiment and theory show that it behaves chaotically at the classical limit although quantum effects tend to suppress the chaos. The result is that among the chaotic trajectories there is a ghostly skeleton of infinitely many periodic orbits, leading to a complex and intriguing sequence of quantum energy levels.

Here, Berry made a remarkable discovery which elegantly connects with one of the central problems in mathematics - described by Berry as the music of prime numbers. The 19th-century German mathematician, Georg Bernhard Riemann, discovered a mysterious hidden structure in the density of primes in the rising number sequence which he defined using a quantity called the Riemann zeta



Coloured caustics

White light passing through a tiny irregularity on a bathroom window is focused into a pattern of caustics softened by rainbow interference fringes. Michael Berry has been studying these delicate motifs on a computer.

function. There are fluctuations in the density which are encoded in a series of frequencies, or magic numbers, called the Riemann zeros. Bizarrely, these frequencies appear to match the energy levels of the periodic orbits in the quantum equivalent of a chaotic system.

For the past 10 years, there has been a two-way traffic between quantum chaology and number theory, points out Berry. "We've been able to feed results from physics back into mathematics using quantum theory to predict statistical correlations among the Riemann zeros which were then found by computation." Berry thinks that this connection must be indicative of something more profound. "It's exciting to think that these three areas - quantum mechanics, chaos and number theory - which seem to have nothing to do with each other are connected." He believes that the Riemann's zeta function describes some model classical chaotic system which when quantised gives the Riemann zeros. "But we don't know what the system is; it's very frustrating and tantalising," sighs Berry.

Although all Berry's work, whether quantum physics or coloured caustics, has revolved around physical asymptotics, he claims that he doesn't work to a grand research plan. Berry's wide interests have arisen naturally because he believes that asymptotics are a powerful tool for dealing with the borders between different areas in physics where one theory approximates to another. "They are much more important than people realise," says Berry. At this moment, Tracie announces that the colour printer has arrived; Berry is relieved and it marks the time for me to leave. But before I go, I ask Sir Michael one more question: would he still do physics if he were starting again now? "Yes I think so, although I could have been an architect or an engineer, but I think that choosing physics has been a lucky hit for me."

Nina Hall