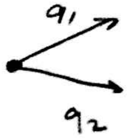
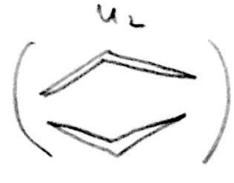
2/20/14

$\underline{q_1 \neq q_2}$ (but approximately)



Types of contributions

u_1 u_2 u_3 ← most divergent



$q_1 q_2 \bar{q}_1 \bar{q}_2$ (4×2)

$q_1 q_2 \bar{q}_2 \bar{q}_1$ (4×2)

$q_1 \bar{q}_1 q_2 \bar{q}_2$ (4×2)

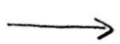
Total # of terms

$\underbrace{4\times 3\times 2}_{24}$ (4 ways to place q_1
3 ways place q_2
& 2 ways $\bar{q}_1\bar{q}_2$ & $\bar{q}_2\bar{q}_1$)

u_1 & u_2 approach each other as $q_1 \to q_2$

$(8u_1 + 8u_2 + 8u_3)\,|\rho_{q_1}|^2\,|\rho_{q_2}|^2$

$\underline{q_1 = q_2 = q}$



Types of contributions:

u_{12} u_3

$q q \bar{q} \bar{q}$ 4

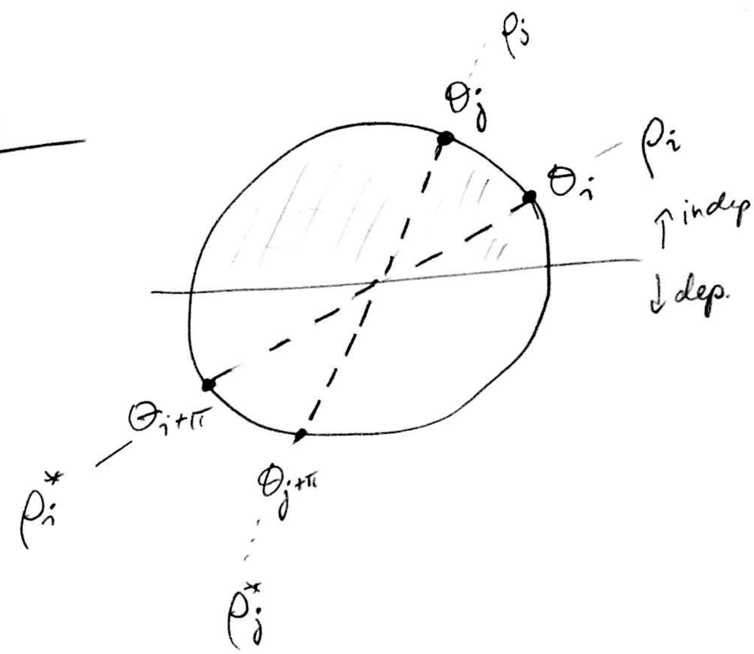
$q \bar{q} q \bar{q}$ 2

Total # of terms

$\underbrace{\dfrac{4\times 3}{2}}_{6}$ (4 ways to place q (1^{st})
3 ways to place q (2^{nd})
divided by 2 (indisting))

$(4u_{12} + 2u_3)\,|\rho_q|^4$

Calculating Free energy



$$\Delta\Omega^{(4)} = \sum_{i<j} 8(u_1+u_2+u_3)|_{\alpha_{ij}} |\rho_i|^2 |\rho_j|^2$$ ← sum over all distinct pairs.

$$+\sum_i 2(2u_{1,2}+u_3)|_0 \; |\rho_i|^4$$

$\alpha_{ij} = \theta_i - \theta_j$

~~Note that in the sum both i,j & j,i have to be included.~~ Alternatively can write as

$$\Delta\Omega^{(4)} = 8\cancel{16}\sum_{i<j}(u_1+u_2+u_3)_{\alpha_{ij}} |\rho_i|^2 |\rho_j|^2$$

$$+2\sum_i (2u_1+u_3)|_0 \; |\rho_i|^2$$

Thus repulsion between nearby density components ($\alpha_{ij} \approx 0$) is 8 times stronger than self-repulsion.
(suppresses clustering).

1

$$F = \sum_i r|\rho_i|^2 + \sum_{i\neq j} u(d_{ij})|\rho_i|^2|\rho_j|^2 + \frac{u_0}{2}\sum_i |\rho_i|^4$$

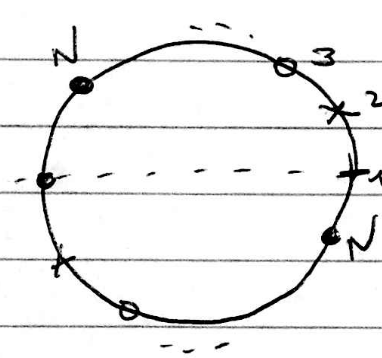
$$\frac{\delta F}{\delta|\rho_i|^2} = 2\sum_{j\neq i} u(d_{ij})|\rho_j|^2 + u_0|\rho_i|^2 + r = 0$$

assume same $|\rho_i|^2 \equiv \rho_0^2$

$$\rho_0^2\left(2\sum_j u(d_{ij}) - u_0\right) + r = 0$$

Penrose $N=5$

($2N=10$ fold diffraction)



$$\rho_0^2 = \frac{-r}{2\sum_{k=0}^{N-1}{}' u\left(\frac{k\pi}{N}\right) - u(0)}$$

Alternative route: express F in ρ_0^2 right away.
does this make sense?

$$F = N r \rho_0^2 + N\left(\widetilde{\sum u}\right)\rho_0^4 + \frac{u_0}{2}N\rho_0^4$$

$$\frac{\delta F}{\delta \rho_0^2} = Nr + 2N\left(\widetilde{\sum u}\right)\rho_0^4 + N u(0)\rho_0^2$$

↪ same result.

(2) $$F = N r \rho_0^2 + N\left(\tilde{\sum} u + \frac{u_0}{2}\right)\rho_0^4$$

$$F = -\frac{N r^2}{2\sum_{k=0}^{N-1} u\left(\frac{k\pi}{N}\right) - u(0)}$$

if $u(\alpha) = u(0) \rightarrow$ const

then $F = -\dfrac{N r^2}{(2N-1)\,u(0)}$ $\leftarrow$ Mermin res

$$= -\frac{r^2}{\left(2 - \frac{1}{N}\right) u(0)}$$

$N = 1$ is the best

But for more general functions $\rightarrow$ No.

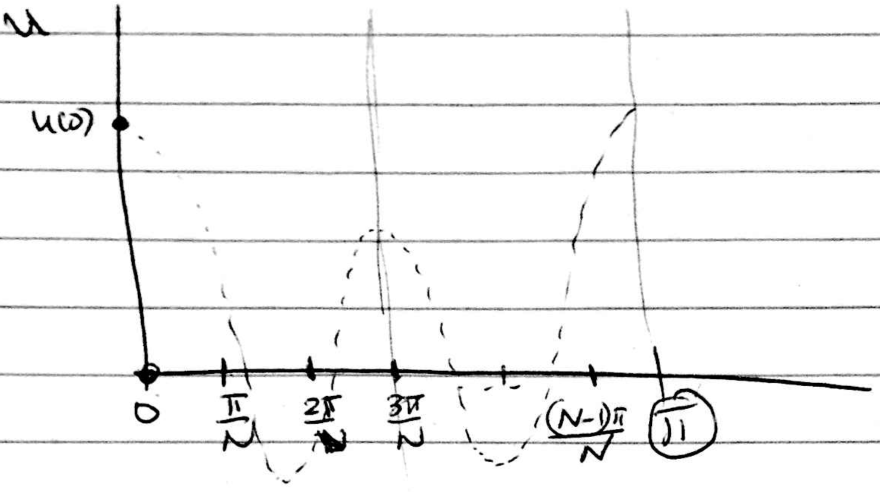
$$F = -\frac{r^2}{\underbrace{\frac{2}{\pi}\left[\frac{\pi}{N}\sum_{k=0}^{N-1} u\left(\frac{k\pi}{N}\right)\right]}_{\substack{\text{integral of } u \\ \text{converges as } N\to\infty}} - \frac{u(0)}{N}}$$

$\frac{u(0)}{N}$ shrinks as $N \to \infty$

N is most favored

in stable systems $(u(0) > 0)$

③



Procedure:

1) Calculate the curve $u(\alpha)$
2) Calculate energy F_N, minimize over N at diff temperatures.

→ Find where 5-fold wins in 2D

3) Effect of cubic terms
4) Effect of fluctuations