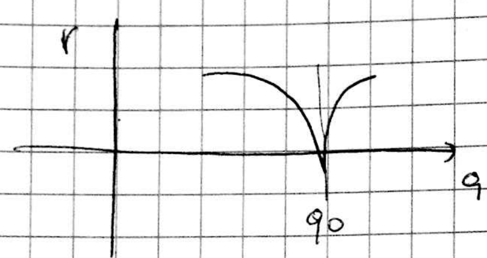
Infinitely stiff Ginzburg-Landau for crystallization

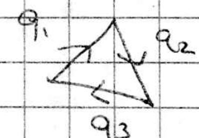
$$\mathcal{F} = r(q)|\rho_q|^2 + \underbrace{v}_{f(q_i)}\,\rho_{q_1}\rho_{q_2}\rho_{q_3} + \underbrace{u}_{f(q_i)}|\rho_{q_1}|^2|\rho_{q_2}|^2$$



For inf stiff only need to keep $|q| = q_0$

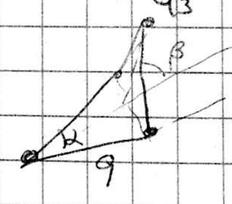
→ all ordering vectors are on a ring/sphere.

Then, for cubic term



$v = v(q_0)$

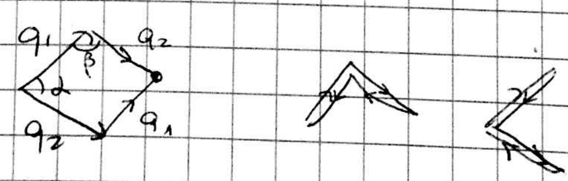
quartic term



$u = u(\alpha, \beta)$

in 2D $u = u(\alpha)$

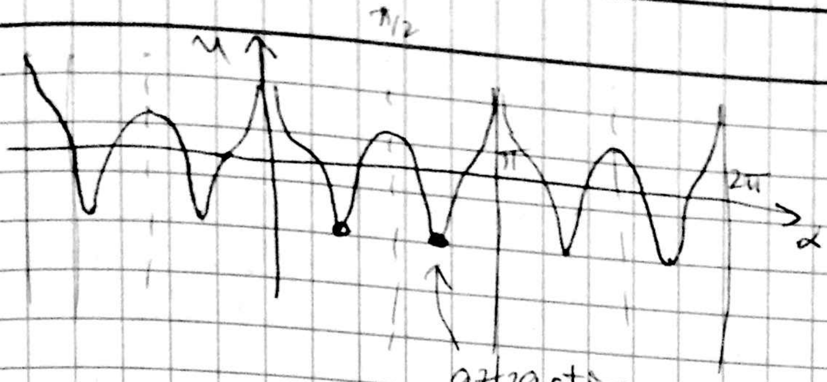
Do 2D first. Symmetries



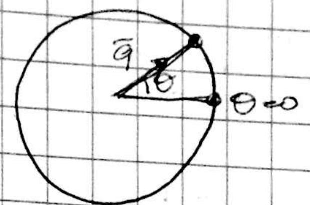
$\beta = \pi - \alpha$

$$u(\alpha) = u(\pi - \alpha)\quad , \quad u(\alpha) = u(-\alpha)$$

$$u(-\alpha) = u(\pi - \alpha) \rightarrow \text{Period } \pi$$



$$\boxed{\begin{aligned} \rho_q &\to \rho_\theta \\ \rho_{-q} &\to \rho_{\theta+\pi} \end{aligned}}$$



$q \parallel \hat{x}$

$$F = r(q_0)\,|\rho_\theta|^2 + v(q_0)\,\rho_{\theta_1}\rho_{\theta_2}\rho_{\theta_3} + u(\alpha)\,|\rho_\theta|^2|\rho_{\theta+\alpha}|^2$$

$$\frac{\delta F}{\delta \rho_\theta} = r(q_0)\,\rho_{\theta+\pi} + v(q_0)\,\rho_{\theta+\frac{2\pi}{3}}\,\rho_{\theta+\frac{4\pi}{3}}$$
$$+ u(\alpha)\,\rho_{\theta+\pi}\,|\rho_{\theta+\alpha}|^2$$

EOM

$$\dot{\rho}_{\theta+\pi} = \alpha \frac{\delta F}{\delta \rho_\theta}$$

or $\dot{\rho}_\theta = \alpha \dfrac{\delta F}{\delta \rho_{\theta+\pi}}$

$$\gamma\,\dot{\rho}_\theta = r(q_0)\,\rho_\theta + v(q_0)\,\rho_{\theta-\frac{\pi}{3}}\,\rho_{\theta+\frac{\pi}{3}}$$
$$+ u(\alpha)\,\rho_\theta\,|\underbrace{\rho_{\theta+\pi+\alpha}}_{}|^2 \to |\rho_{\theta+\alpha}|^2$$