Reasonable short-run decision making under uncertainty

Mihail Turlakov[1]

Abstract

Formulate more practical version of Haghani-Dewey experiment

Develop and synthesize Browne-Brown strategies. **Propose and test various heuristics for short-run.** Understand the differences between short-run and long-run Kelly strategies, the differences between ensemble average and time-series average.

Discuss and analyse strategies and their upsides and downsides with corresponding behavioural biases. Explore the properties and the phase space of trend (autocorrelation) and carry (stable edge) strategies. Lay the foundations for the proprietary first-principles trading and investment strategies.

Discuss and understand the differences between traders, investors, and gamblers. Rebalancing and liquidity.

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A simple and fascinating experiment on a biased coin throws new light on the foundational problems of finance theory and practical decision-making. Victor Haghani and Richard Dewey performed and discussed a basic foundational experiment in the recent paper *"Rational Decision Making under Uncertainty: Observed Betting Patterns on a Biased Coin"*. Motivated by the experiment, several authors, A. Brown, V. Ragulin, and A. Viswanathan, explored different approaches and solutions of the problem independently.

New Haghani-Dewey experiment with uncertain bias

In the original Haghani-Dewey experiment, the subjects were given 25$ and offered the opportunity to bet on biased coin with 60% chance of heads at even payout. A particular uncertainty (unknown) was that a maximum payout for the whole game would be revealed only if the subject tried to place a bet, which could allow the total payout to exceed the maximum cap.

I would like to propose a new version of the experiment, which has closer similarities to practical decision making in the financial markets. In particular, the bet is now offered on the *uncertain bias* coin. Namely, the subjects are told that the coin is heads biased but the bias probability is unknown. The number of flips is fixed and not large. No explicit maximum cap might be necessary, since the fixed number of flips caps the maximum payout with the bet size always limited by the current wealth, which can be sufficiently small at the start (no margin borrowing is permitted).

The main motivation of this new experiment is to explore the role of uncertainty rather than risk in the sense of (Knight 1964). Besides the uncertainty of the maximum payout, the original Haghani-Dewey experiment is the decision making under risk, since the probabilities of a biased coin are known. Therefore, the expectation (edge) and variation (variance) can be calculated. In the new experiment, we can assume "weak uncertainty", which assumes a prior for the probability density of heads, and "strong uncertainty", no prior and frequentist approach. A "super-strong uncertainty" would allow fully unpredictable outcomes, in the sense of unknown unknowns of Donald Rumsfeld, beyond the confines of a simple game. The case of "super-strong uncertainty" is beyond this paper.

The main interest of this and related research is the position sizing of the bets, which is also called money management and risk management in different contexts. In the markets, two key parts of the decision making are deciding on the probabilities and outcomes and deciding on the sizing of the bets. In this research, we clearly focus only on the strategy of the sizing of sequential bets.

Importantly, we would like to explore and recognize the most robust and practical tactics and strategies rather than mathematically precise solutions (and therefore likely limited applications), so that practical application in live and possibly tense situations is natural and effective. This means that only basic calculations would be needed for the decision making in the real situations.

Short-run and long-run defined

In the conventional context of the known biased coin, let us define the fixed bias probability p of heads and the number of flips N. The positive expectation (edge) and the variance are $\mu = 2p - 1$ and $Variance = \sigma^2 = 4p(1-p)$, correspondingly.

For the well-known limit of the very large number of flips, Kelly criterion shows that strictly optimal betting[2] is the constant proportion betting $f^* = 2p - 1$. The rate of wealth growth is $\exp(\mu\, f^*\, N)$. For a recent detailed review of Kelly criterion and Samuelson's objections, see the paper (Ziemba 2015).

It is important to distinguish between short-run and long-run in the number of flips, while most discussions of Kelly criterion do not recognize this point. The crossover number of flips between short-run and long-run is $\mu\, f^*\, N_{cross} \sim 1$ (see also (Peters 2011)), therefore $N_{cross} \approx \frac{1}{(2p-1)^2}$, which is 25 flips for *p=0.6*. The short-run is when the number of flips is smaller than N_{cross}. The long-run is when Kelly criterion becomes applicable and the number of flips is much larger than N_{cross}.

Interestingly, in the continuous limit $N_{cross} \approx \frac{1}{Sharpe^2}$, which is 10-25 years for typical asset classes with Sharpe[3] ratios=0.2-0.3 in financial markets. So practical strategies are necessarily short-run.

Let us define N_{risk}, which separates the regimes of uncertainty and risk. N_{risk} is the number of flips which allow to determine the bias probability by using Bayes theorem. Depending on various parameters N_{risk} and N_{cross}, there might exist multiple regimes. Actually, a generic argument shows that N_{risk} is about the same number as N_{cross}. $N_{cross} \sim N_{risk}$. This is because to resolve the biased centre of Gaussian distribution, we need the width to be narrower than the distance away from the unbiased middle, i.e. $\frac{1}{\sqrt{N_{risk}}} \sim \left(p - \frac{1}{2}\right)$. Therefore, there are only two qualitative regimes of the fixed biased coin game.

Uncertainty and short-run regime $N \ll N_{risk}$: This regime appears to be addressed in Brown's approach, but perhaps the convexity of time-series leverage might be exploited better. Better control of drawdowns and smoother expectations are of value also.

Risk and long-run regime, Kelly limit: $N_{risk} \ll N$. This is the well-known long-run of Kelly strategy dominance. The uncertain bias becomes eventually certain after many bets ascertaining the bias probability, and therefore Kelly criterion will apply.

The questions of long-run versus short-run are interesting from several perspectives. First, for the case of a single bet there is no other way but to use utility theory. For the long-run, importantly if and only if IID time-series, log-utility is optimal (Ziemba 2015). Second, long-run and short-run can clarify further the differences between ensemble-averaging and time-averaging limits (Peters 2011). Third, other solutions than Kelly strategy can be more practical for short-run as well as controlling the drawdowns. The new results are explored for the short-run, while the long-run has a well-known Kelly-Thorp solution.

Kelly convexity

In this section, I review the salient properties and qualitative insights of Kelly criterion.

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[2] Interestingly, the continuous limit solution is slightly different $f^* = \frac{\mu}{\sigma^2} = \frac{2p-1}{4p(1-p)}$, since $4p(1-p) = 0.96$ for p=0.6 is close but different from 1.

[3] Sharpe ratio is $Sharpe = \frac{\mu}{\sigma}$

We can compare several quantities. A single and extremely lucky outcome is $V_{lucky} = W * 2^{300}$. The expectational value or ensemble average is $V_{ensemble} = W * (0.6 * 2 + 0.4 * 0)^{300} = W * 1.2^{300}$ is huge if the full wealth is invested in St Petersburg's paradox style. The median value of the unmanaged time-series is $V_{median} = W * (1.2^{0.6} * 0.8^{0.4})^{300} = W * e^{0.6ln1.2+0.4ln0.8^{300}}$. The optimal Kelly final wealth is $V_{Kelly} = W * (0.6 * 1.2 + 0.4 * 0.8)^{300} = W * 1.04^{300}$. Notice that only in the second order of expansion $e^{0.6ln1.2+0.4ln0.8} > 1.04$, because $0.6ln1.2 + 0.4ln0.8 = 2.01 * 10^{-2} < \ln(1.04) = 3.92 * 10^{-2}$. Clearly, we have $V_{median} \ll V_{Kelly} \ll V_{ensemble} \ll V_{lucky}$. Due to rebalancing and optimisation, Kelly growth is significantly better than simple time-series, but Kelly growth cannot achieve inaccessible ensemble average. Ensemble average was criticised at length by Peters-Gell-Mann.

Browne strategy

Kelly criterion is a major theoretical insight, which provides a practical guideline,

The specific problem is p between 0.45 and 0.65 with the average p=0.55. What is the solution with utility theory? What is optimal convexity in the sense of Kelly convexity?

A) The tradeoff between „model bias"/underfitting and „model variance"/overfitting
B) Or the solution in the sense of Gigerenzer of the balance between bias and variance, not accuracy-effort tradeoff. Less-is-more (Gigerenzer) and more-is-different (Anderson), then less-is-different **meaning short run.**

Drawdown can depend differently on the tails/loss probability and the tail outcome (uneven bets).

Breaking long-run into short-runs according to Herbert Simon arguments of the complexity of the hierarchies by analogy with the watchmakers.

Optimal frontier for time-series of a single asset

Trend and mean-reversion extremes

Diversification and portfolio theory for time-series

Getting earlier to the long-run Kelly by diversification, see Peters's paper.

Any insight about diversification from Aaron Brown.

Notice also a discussion about time diversification and portfolio diversification in the context of general utility functions.

Conclusion

The leverage and the uncertainty have to be managed

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Practical principles of sequential decision making in trading and investing

Based on the technical results, I would like to review the practical decision-making in financial markets.

Behavioural biases

“Edge is the key” is the main principle of risk taking. In the world of the quantifiable risk, Kelly criterion builds on the amplification of the risk premium (edge). Consistent and skilful management of the uncertainty and the leverage can be the source of structural edge, especially since the market risk premiums can be largely due to the uncertainty (Lemperiere 2015).

Pitfalls of heuristics (Kahneman) versus dual robustness of heuristics (Gigerenzer).

Singular limits

The conceptual almost philosophical problems of the classical mechanics limit of the quantum mechanics (the foundations of quantum mechanics) and “time diversification” puzzle in the finance theory are exactly the same! [[mapping between low-temperature glasses and quantum electrodynamics- was it ruled out due to different interaction vertices?]]

Ensemble-average versus time-series is a spectacular and deep discussion, still related to the understanding of risk and uncertainty by Paul Samuelson. “Singular limits” by Michael Berry sheds nice and accessible insights from several areas of physics and in particular from the limit between classical and quantum mechanics. Another interesting angle is how Murray Gell-

Mann argues about time-series in finance, and yet many-worlds interpretation of quantum mechanics, which is in the style of ensemble-average. Utility theory is "symmetry breaking" in Anderson's condensed matter theorist language, ergodicity breaking. More is different.

Christensen (Christensen 2011) gives a good discussion of Samuelson versus Thorp by underlying the differences between utility theory and time-series limit (see pp. 33-35). One-bet preference and transitivity are important parts of the arguments started by Samuelson. Gell-Mann and Peters analysed the foundations of the utility theory. "Long run" is defined as probability to outperform at certain confidence level. The whole Thorp-Samuelson debate is called "time diversification puzzle" about the relationship between investment risk and investment horizon, and "each side is right on its own terms".

The differences between traders, investors, and gamblers.

The decision making framework and risk management

Liquidity and Leverage

Liquidity and opportunities

Rebalancing and liquidity. Vladimir Ragulin - I believe it depends on how easy it is to rebalance one's portfolio. In the context of a bank loan book rebalancing is extremely costly, so all the classical Samuelson solutions that assume that an investor can continuously rebalance as his wealth and risk-aversion change are not realistic -- so one needs to explicitly model the long-term, as you do. On the other hand, for hedge funds or futures-trading CTA's continuous rebalancing is not such a bad assumpton

Appendices – background discussions

Notes and questions about the paper "Optimal betting strategies with uncertain payout and opportunity limits" by Aaron Brown

The approach is intuitively appealing, robust and practical. First, order the outcomes. Second, assign the highest probability to the maximum outcome and so on, and the lowest probability paths to a zero outcome.

Q1: The paper is called "Optimal betting strategies…". *What quantity is the subject of the optimisation here?* No explicit utility or growth rate are being used. I agree fully with the goal and the intention of "*…having simple, reliable general tools to make reasonable decisions" and "… actually trusting and using those tools".*

The information edge of the known probability *p* and the edge *(2p-1)* are not used in the strategy by construction or explicitly. Thus it is strange to find on the page 10 at the bottom of the 1st paragraph "The surprise, if any, is that *p* does not affect bet size in the certainty case". At least in the sense of information theory, Brown strategy is not optimal, clearly, Kelly exploits the edge *(2p-1)* to the fullest. It seems to me that the bias probability is assumed unknown but higher than 50%.

There seems to be path-averaging (equivalently, ensemble-averaging) logic used implicitly throughout the paper. Or is it risk-neutral or utility based logic here? This assumption/constraint does not look correct to me, since the 2^N outcomes are the result of *sequential decisions rather than simultaneous utility-based decisions.* One of the main points is that each path matters, and this path can easily be the only path taken in reality. So it is better to avoid making many, if any at all, paths to pay zero total payout.

Another point is that the target payouts need to be reasonable and achievable. For instance, for 10 flips and 1$ initial wealth, and the maximum payout is 1024$, is the strategy still to assign 1 path to 1024$ and the rest in the same spirit of the paper? For 20 flips, still using the same assignment of paths?

Yet for small number of bets, it does seem to make sense that the bets can be larger than Kelly constant fraction due to bets number discreteness.

Q2: *what is the calculation for the quantity r in the section "According to Kelly" (page 5)? How to arrive to the equation for "The exact adjusted Kelly bet requires solving for r …."?* Somehow based on "… to adjust Kelly solution is to minimize the percentile of the distribution that gets the maximum outcome". The approximate *r* is given to be $r = \sqrt{\frac{2\ln(\frac{M}{W})}{n}}$.

One approach, which comes to mind, is to find the fraction *f* corresponding to the median path yet $((1+f)^p(1-f)^{1-p})^n = \frac{M}{W}$. We have $pln(1+f) + (1-p)\ln(1-f) = \frac{\ln\frac{M}{W}}{n}$. Equivalently, $f^2 - 2(2p-1)f + \frac{2ln\frac{M}{W}}{n} = 0$. Two solutions are $f_\infty^* = (2p-1)$ and $f_n^* = \frac{2\ln\frac{M}{W}}{n\,(2p-1)}$ assuming that $(2p-1) \gg \frac{\ln(\frac{M}{W})}{n}$, which is perhaps not following the idea of Brown strategy.

Another approach to estimate the optimal rate r_0 with the quadratic dependence at the maximum[4] (without the proof of the maximum) for the exponential growth $\exp(r_0^2\, n) = \frac{M}{W}$, which makes $r_0 = \sqrt{\frac{\ln(\frac{M}{W})}{n}}$.

Q3: are Figures 6-8 for a first bet for the uncertainty case? For instance, for 2nd and 3rd bets, different figures and different Expected Cap should be calculated? Essentially, for Figure 6, it

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[4] The exponential optimal growth rate of the wealth is $e^{\frac{\mu^2}{\sigma^2}N}$ according to Kelly criterion.

is not clear how for small bet $1 and initial wealth 1$, the maximum cap of 32$ can ever be reached.

Q4: what is the origin of the number 10.54$ on the page 8 in the 3rd paragraph? This number did not appear anywhere in the text before it seems.

Q3: *why is the constraint?* "…and the only constraint on choosing them (paths) is that they average to your initial bankroll."

This constraint is introduced in the recent paper (A. Brown 2017) and the earlier one (A. Brown 2005), yet this essential constraint remains unjustified and unexplained, especiallyin the context of a trading organisation. Perhaps, this constraint corresponds simply to the initial assumption of even outcome betting before weighting in the biased probabilities. More generally, the outcomes can be uneven and contribute to the possible edge.

Notes and questions about the paper "Got to be in it to win it" by Vladimir Ragulin

The paper explores several forms of utility functions and risk aversions in the original Haghani-Dewey experiment.

Q1: *Is it possible to observe the dominance of log-utility player 2 in the infinite cap case for Expected Payoff (Table 2)?* One should observe the transition of the dominance from player 2 to player 1 by moving the cap payout from infinity to the finite values. Initially player 2 should have higher Expected Payoff by Kelly-Thorp for the infinite cap, and later player 1 (linear utility) becomes favoured in Expected Payoff (not applying utility). This is despite each player optimises the corresponding utility. Player 1 should always have highest Game-Certainty-Equivalent indeed.

The expected value of 300 flips is $25\$ * 1.04^{300} = 3.22 mio\ \$$. *It seems that there is a very strong effect of the cap even for the cap of* $10mio$\$. See the recent summary of Kelly strategy and comparison of various utility players in (Ziemba 2015). Player 2 (log-utility) should dominate in Expected Payout, no matter what, relative to player 1(linear utility) and player 3 (risk-averse CRRA). This is Kelly-Thorp result in the long-run and infinite cap case.

Q2: is there a proof based on Jensen's inequality that player 1 is *always* dominant for Expected Payout? From the initial reaction by Vladimir

Q3: are the results of Ragulin and Viswanathan the same in all details?

Q4: How can short-run be different from long-run from the utility theory perspective, including the developments of Epstein-Zin? Simple utility functions do not distinguish between time-series-averaging and ensemble-averaging.

Some Clarity on Risk Parity: Victor Haghani and James White

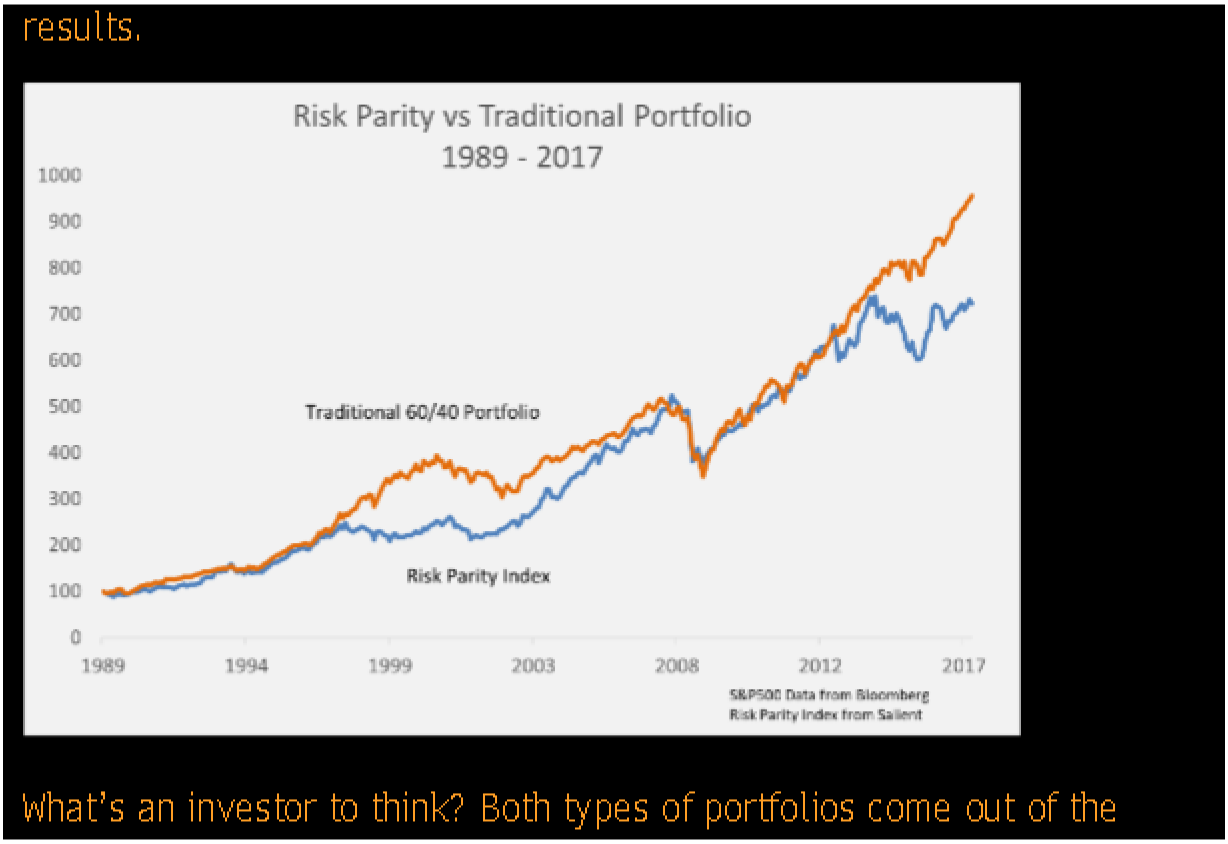
Risk parity and traditional portfolios are usually presented as being philosophically miles apart, but they are actually much closer.

By Victor Haghani and James White

(Bloomberg Prophets) --

Google “risk parity” and you’ll see a grab bag of conflicting results: articles and posts trying to explain what it means, why it reduces stock-market volatility, why it increases stock-market volatility, why it’s less risky than a traditional portfolio, or why it’s more risky, among other things.[1] **We’ll try to cut through this confusion to show that risk parity and traditional portfolios are closely related in philosophy.**

Risk parity is all about how an investor allocates **risk, not capital, typically with the use of leverage and with the idea that an equal risk allocation** to various asset classes increases the benefits of diversification. Risk parity and traditional portfolios are usually presented as being philosophically miles apart, and hard to compare or analyze side by side except by looking at the historical record, such as in the chart below. **The problem, though, is that financial market history is limited in that it reflects only a very specific** set of historical conditions and, as we know, past performance isn’t indicative of future results.



What’s an investor to think? Both types of portfolios come out of the same theory of portfolio construction, but with different sets of basic starting assumptions. The theoretical toolkit we’re talking about here is the “Optimal Expected Utility” framework applied to financial markets by Paul Samuelson and his protégé Robert Merton starting in the 1960s.[2] Their work helped them both garner a Nobel Prize, and produced a set of practical tools for determining how much of one’s wealth should be allocated to different investments with the understanding that the future is uncertain. The tools are primarily based on an investment’s expected excess return, volatility, and the investor’s personal level of risk-aversion.[3]

The basic idea is that an investor's utility doesn't keep going up as investment size -- and thus risk -- increases, but instead there's an optimal investment size that maximizes expected utility given one's personal level of risk-aversion. This simple idea leads to some powerful results. With the five assumptions below, the utility toolkit tells us that the portfolio that maximizes expected utility is the one with the highest Sharpe ratio -- a common measure of risk-adjusted returns4 -- levered or delevered to an optimal level of risk.

Assumptions for Optimality of **Basic Risk Parity Portfolio**

- All assets follow a random walk and are continually tradeable
- All assets have zero correlation with each other
- All assets have equal Sharpe ratio over a long horizon[5]
- Unlimited leverage is available at the risk-free rate
- No fees, transactions costs, or other drags on return

To build that portfolio with uncorrelated, equal-Sharpe ratio assets, we'd hold an amount of each asset that is inversely proportional to its volatility, resulting in each asset contributing an equal amount of risk to the portfolio. This is why it's called "risk parity" investing.[6] Using some stylized risk/return assumptions, the table below shows how this works with two risky assets for an investor with a "typical" amount of risk aversion.[7] The first three rows show arbitrary allocations, and the final three rows show utility-optimal allocations corresponding to the given portfolio assumptions.

Portfolio	Stocks	Bonds	Expected Excess Return	Risk	Sharpe Ratio	Risk-adjusted Return
Stocks	100%	0%	4.0%	16.0%	0.25	0.2%
Bonds	0%	100%	1.0%	4.0%	0.25	0.8%
Traditional	60%	40%	2.8%	9.7%	0.29	1.4%
Risk Parity Unlevered	20%	80%	1.6%	4.5%	0.35	1.3%
Risk Parity Levered	50%	210%	4.1%	11.6%	0.35	2.1%
Risk Parity+ 0.6% Extra Borrow Cost	45%	85%	2.5%	8.0%	0.31	1.5%

The Merton toolkit suggests our investor would optimally want to own, via leverage, $260 of the equal-risk portfolio for every $100 of savings, resulting in the "Risk Parity Levered" portfolio in the table. The performance of this portfolio is quite a bit better on a risk-adjusted basis, 0.7 percent a year to be exact, than the traditional 60/40 stock/bond portfolio.[8]

We made some strong assumptions to get this result. Let's see what happens when we loosen just one of them and assume that leverage isn't available at the risk-free rate, but at a rate 0.6 percent higher? That cuts the risk-adjusted return of the optimal portfolio to 1.5 percent per year. This portfolio is very close, both in risk-adjusted return and Sharpe ratio, to the traditional 60/40 portfolio, making the traditional portfolio functionally optimal given the assumptions. We get the same result if we assume the investor doesn't want to use leverage, regardless of the rate. Changing this assumption isn't some abstract technicality: **Leverage in real markets is not freely available at all times or at consistent rates, and there are many reasons an investor may choose to eschew leverage.[9]**

The portfolios we see here represent **two ends of a spectrum -- but the range is surprisingly narrow**. In our admittedly stylized two-asset example, only 0.7 percent per year of risk-adjusted return separates the fully-levered risk parity portfolio from the unlevered traditional one, **which gives a sense for the level of fees, trading costs and extra borrowing expense a risk parity strategy could plausibly support.**10 If the five assumptions above seem reasonable, risk parity portfolios may make sense for you, but if not, a more **traditional portfolio may be a better fit, and is just as consistent with good finance theory.**[11]

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1. 1 There are a number of overviews of risk parity online. Here's one: https://www.aqr.com/-/media/files/papers/understanding-risk-parity.pdf

2. 2 One of the most seminal papers is "Robert C. Merton. Lifetime Portfolio Selection under Uncertainty: the Continuous-Time Case." The Review of Economics and Statistics (51), 1969.

3. 3 A core result is that, for one risky asset following a random walk, the optimal investment size is where μ -r is the asset's excess return, σ its volatility, and n the investor's coefficient of risk-aversion. We often refer to this result as the "Merton Rule."

4. 4 Sharpe Ratio is the ratio of an asset's excess return to volatility: SR =

5. 5 This is consistent with both the historical record and what many equilibrium models would predict.

6. 6 Assuming risk and volatility are interchangeable for the purposes of this discussion. For uncorrelated assets with different Sharpe ratios, the solution is to scale each proportional to Sharpe ratio and inversely proportional to risk.

7. 7 We'll define typical here as that degree of risk aversion that would maximize expected utility by investing 100 percent of savings in a stock/bond portfolio with a 60/40 mix. With the numbers in our illustration, this implies a coefficient of risk aversion in the Merton model of 3. For readers familiar with the Kelly Criterion, this means our investor is 3 times as risk averse as a Kelly bettor. Also, typical risk parity implementations include four or more assets, including commodities and credit.

8. 8 Risk-adjusted return = Expected Return – ½ * * , where is the coefficient of risk aversion

9. 9 A few reasons that come to mind: 1) real markets may not follow pure random walks but can also gap, 2) leverage may not be easily adjusted once set, 3) terms other than rate may not be attractive, or 4) whenever the investor hears the word "leverage" he or she suffers painful flashbacks.

10. 10 And they are separated by only 0.06 in terms of Sharpe ratio, although some back-tests suggest a difference of close to 0.2. As discussed in "What's Past is Not Prologue," we cannot rely on historical data on its own to support or reject the existence of this amount of difference in Sharpe ratio.

11. 11 The authors would like to thank Larry Hilibrand and Vlad Ragulin for their help.

Humans Have an Essential (Small) Role in Markets: Aaron Brown

Until computers learn fairness, or the human race abandons the ideal, humans are essential in the investment process.

By Aaron Brown

(Bloomberg Prophets) --

I recently participated in a debate among hedge fund risk managers on a panel titled "This house believes that qualitative and human elements in investment are redundant."[1] The premise was to have three speakers agreeing with that resolution, and three disagreeing. But instead of two distinct sides, it quickly became apparent that each of the six participants had a different opinion. In order from "Most For" to "Most Against,"[2] the arguments were:

- Humans should lie on the beach while computers run the financial system.
- The only role for humans is to pull the plug if computers screw up.
- Humans will account for only 1 percent of the investment process in the future.
- Humans have an essential small role in investments (that was me).
- Humans beat computers at things like negotiating restructurings.
- Computers are only a tool used by some humans.

I base my position on three facts and two speculations.

1. Your unconscious brain is far superior to the best computer. The most sophisticated computers have only recently learned to tell dogs from cats in clear, still pictures; and only most of the time. Your brain effortlessly parses a confusing visual field in real time for

thousands of more difficult distinctions while simultaneously doing the same for other senses and internal processing, with virtually no important errors. And no one taught it how.

2. Your conscious brain is far inferior to the worst computer. It quails at multiplying two three-digit numbers. This is why your unconscious brain gets the important jobs, like regulating your body and deciding when to fight and when to run, while your conscious brain hums advertising jingles and wonders which celebrity marriages are in trouble.

3. You make most -- possibly all -- decisions before your conscious brain is aware of them or has been consulted.

4. (Speculation) Consciousness evolved not to improve decisions, but to explain them after the fact to other humans. If your paleolithic ancestor killed Og to get his food or mate, that would make him unpredictable and dangerous to the tribe, and might result in his murder or banishment. He would be treated like a dangerous wild animal. So he evolved a conscious brain to say -- and believe -- "Og blasphemed and I sacrificed him to appease the anger of the Gods." The tribe might or might not accept the plea, but consciousness was like a lawyer to negotiate the consequences of decisions and promote predictable social cohesion.

5. (Speculation) Computers will soon surpass the unconscious brain in making investment decisions thanks to advances in machine learning, better understanding of human decision-making and better understanding of finance. While the brain will retain some advantages for decades, computers can process far more data much faster and cheaper with less interference and, most important, will be designed and evolve for financial objectives.

It is the **weak human conscious brain that will remain essential, not to make investment decisions, but to explain them after the fact**.[3] Until we entrust the entire financial system to Colossus or Skynet, institutional investors, fund managers, dealers, regulators and other entities will have to coordinate. Currently that process is mediated by trust and human legal machinery.

Humans will have to invent new ways to explain the actions of their computers, so that other market participants can predict and trust them enough to allow trading. To date, computers have not done well at this task, while your conscious brain has been honed by millions of years of evolution to explain decisions it does not understand. It's not much of a change when those decisions are from an external black box rather than an internal unconscious brain.

This is not a trivial task; it is difficult and essential. No human or computer has much idea of what an optimal set of prices should be -- if there even is one. **The financial system exists to set consensus values on things no one knows how to value,** so the economy can move forward in an unpredictable world. It would be nice if these values were accurate, [4] but it is essential that they are fair. People will only entrust their savings and their businesses if the game is not rigged. **Until computers learn fairness, or the human race abandons the ideal, humans are essential in the investment process.**

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1. 1 The debate was sponsored by Risk magazine. Stuart MacDonald of Bride Valley referred, with Rayne Gaisford (Folger Hill), Bob Savage (CC Track Solutions) and Efrem Sternbach (Paulson) For and Thomas Zucosky (Discovery), Amy Wierenga (Blue Mountain) and me Against.
2. 2 I won't identify the positions with the people, because these are my quick summaries of my understanding of their positions, not necessarily how they would describe their views themselves.
3. 3 So when a computer loses 50 percent of the asset value it controls and causes a flash crash, there will be humans around to say -- and believe -- "The computer anticipated and prevented a Black Swan that would have caused greater loss and disruption." Other participants may or may not accept this plea, but without a defense the computer would have to be shut down. If computers are shut down every time they do something hard to understand and apparently bad, they will soon be eliminated from the financial system.
4. 4 Fischer Black famously defined an efficient market as one that got prices right within a factor of two some 90 percent of the time. I told him he should write half the time and almost convinced him. He was a guy who really cared about precision even in broad brush statements.

Why Thaler's Practical Contribution Matters: Mohamed A. El-Erian

He applied insights from psychology and neuroscience to a discipline that oversimplified human behaviors.

By Mohamed A. El-Erian

(Bloomberg View) --

The Nobel Committee did well in awarding this year's prize in economics to Richard Thaler. The University of Chicago scholar has made pioneering contributions to behavioral economics and finance that have been both influential and, in terms of practical applications, consequential and welfare-enhancing.

Overcoming several challenges from an economics profession too focused on elegant but unrealistic approaches, his work has direct relevance to observable outcomes and decision-making. That is one of many reasons why policy makers and investors should read Thaler's accessible and engaging work, especially given today's fluid global economy and unusual market dynamics.

Even before the Nobel announcement, behavioral economics and finance -- the application of insights from psychology and neuroscience to a discipline that overly simplified human behaviors and interactions -- have been attracting growing interest, and rightly so.

Thaler and others have **already "nudged" individuals to make better decisions about their financial security, helped regulators better understand the drivers of excessive risk-taking**, and provided investors with a better understanding of the perils of repeating past mistakes. Their work has encouraged governments and companies to make a greater effort to incorporate behavioral science, including elements related to unconscious biases, as well as comfortable but misleading rules of thumb and analytical shortcuts. The prize may also help in the multiyear challenge of restoring the standing and credibility of the economics profession, which took a battering after most economists completely misread the run-up and aftermath of the global financial crisis.

The laureate's work is also highly accessible. This is particularly true of his last two books, "Misbehaving: The Making of Behavioral Economics" and "Nudge: Improving Decisions About Health, Wealth, and Happiness," which contain engaging discussions of behavioral science, including its application to a range of real-world issues. They are must-reads for those who are new to this area of economics, as they help explain today's world of fluidity and unusual uncertainty in economics and politics, in which markets have remuneratively disconnected in a striking low-volatility fashion.