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Information, Games, and Investment Strategies

Mihail Turlakov

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OUTLINE - Information, games, and investment strategies

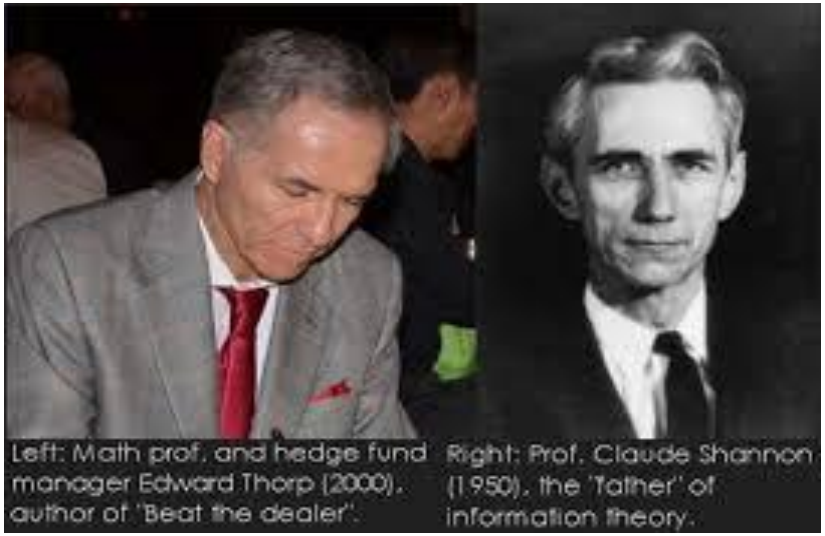
- Markets viewed through information and game theories
  - *Inefficient markets, information asymmetries, and game strategies*
- Kelly criterion
  - *The properties and history of Kelly-Thorp strategy*
- Leverage and Uncertainty (brief summary)
  - *Kelly criterion, tail risk and uncertainty*
  - *Kelly Parity versus Risk Parity and Markowitz*
- Meta-game strategies for the systematic factors (work in progress)
  - *Kelly-Thorp strategy for absolute momentum*
  - *Game theory interactions between basic strategies*

Markets and information theory

…. Noise is contrasted with information
-Fischer Black-

- On the Impossibility of Informationally Efficient Markets [Grossman, Stiglitz, 1980]
  - ✓ "Hence the assumptions that all markets, including that for information, are always in equilibrium and always perfectly arbitraged **are inconsistent when arbitrage is costly**."
  - ✓ The limits of Efficient Markets Hypothesis
  - ✓ **Financial Markets are information systems** – recent fashions and trends - AI, big data, machine and deep learning, and information commoditisation

- **Shannon Information theory and statistical games**
  - ✓ **Kelly strategy and Thorp's success [Thorp, 2017]**


Left: Math prof. and hedge fund manager Edward Thorp (2000), author of 'Beat the dealer'.

Right: Prof. Claude Shannon (1950), the 'Father' of information theory.

Markets and game theory&practice

..noise creates the opportunity to trade profitably, but at the same time makes it difficult to trade profitably.
-Fischer Black-

- Adaptive and efficiently inefficient markets
  - ✓ Adaptation, competition, and evolution – "Adaptive markets" [Andrew Lo, 2017]
  - ✓ "The strategic analysis of financial markets" [Steven Moffitt, 2017]
  - ✓ Economic and transactions machine [Ray Dalio]

- "Epsilon theory" - game theory mind-set - Ben Hunt
  - ✓ Game-theoretic decision making (Common Knowledge, Coordination Game, and Prisoners' Dilemma - Fed, Mr. Market, governments), *QUID news trees* of the information and news

- "The Poker Face of Wall Street", "Red-blooded Risk" – Aaron Brown
  - ✓ Risk management; Uncertainty and noisy information exchange in poker, other games, and life

- "Volatility and the Allegory of the Prisoner's Dilemma" – Christopher Cole
  - ✓ Volatility: the market price of uncertainty – volatility as the most important asset class

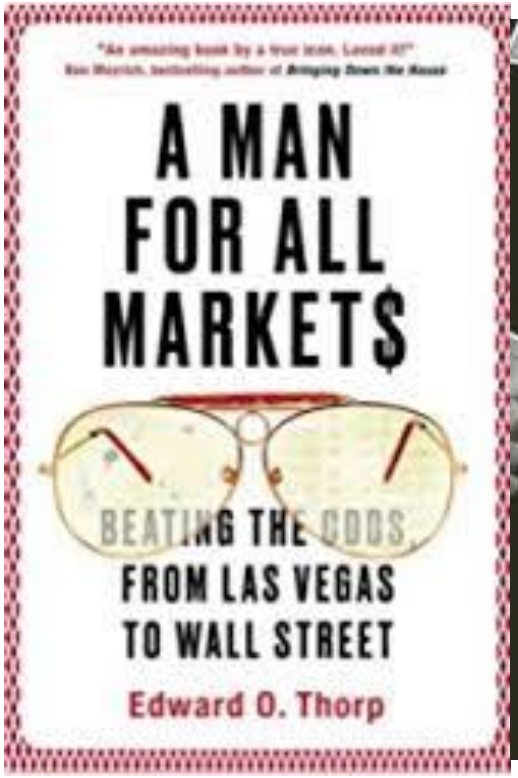
Arc of skill on the border of exploration and exploitation

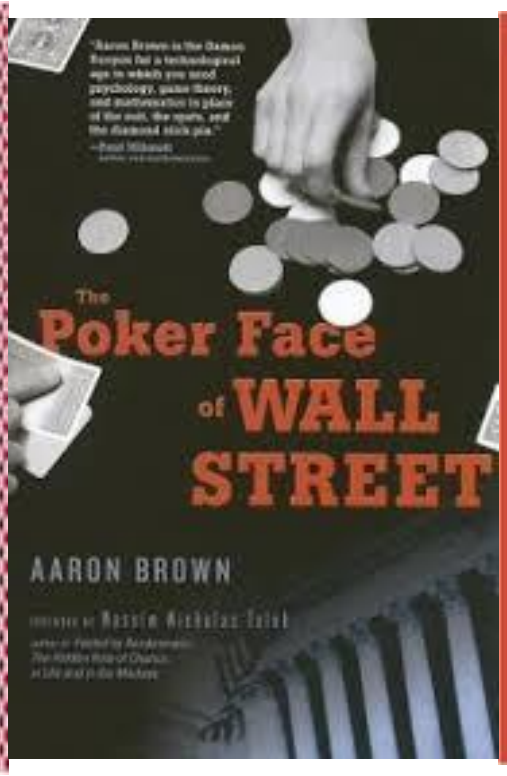
Pursue some path, however narrow and crooked, in which you can walk with love and reverence

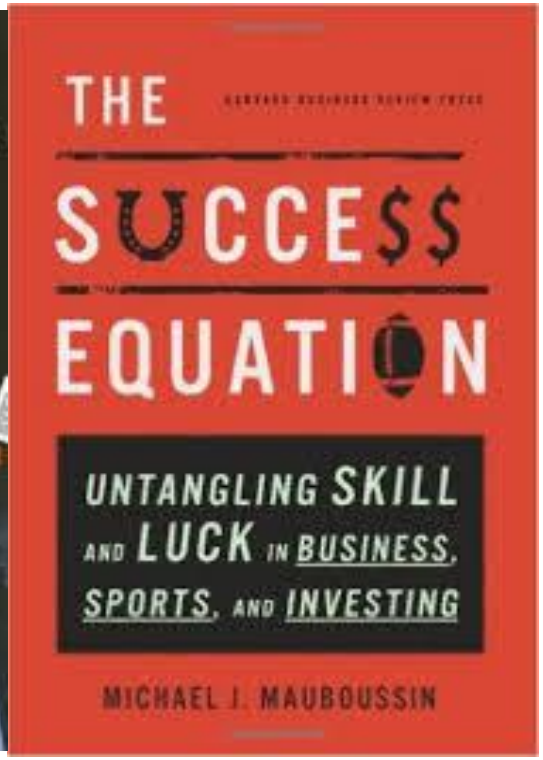
-Henry David Thoreau-

- Finding investment strategies under risk and uncertainty

https://www.linkedin.com/pulse/finding-investment-strategies-under-risk-uncertainty-mihail-turlakov







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Kelly criterion (1) - introduction

- Information theory – Shannon
  - ✓ **Betting on biased-coin flips** – lets call it **Kelly game**
  - ✓ Kelly:"…the maximum exponential rate of growth of the gambler's capital is equal **to the rate of transmission of information**"
  - ✓ For multi-period IID process, Kelly optimization is equivalent to log-utility
- Kelly criterion [Kelly]

$$\text{LEVERAGE} = \frac{EDGE}{ODDS} = \frac{USEFUL\ PRIVATE\ INFORMATION}{NOISE} = \frac{\mu}{\sigma^2}$$

Kelly Criterion

f = fraction of current bankroll to bet

b = net odds: you could win $b (plus $1 bet) for a $1 bet

p = probability of winning

$$f = \frac{bp - (1-p)}{b}$$

- Multiple qualitative interpretations

$$\textbf{LEVERAGE} = SKILL * LUCK$$

$$\textbf{LEVERAGE} = \frac{DIFFUSION\ TIME}{DRIFT\ TIME}$$

Kelly criterion (2) – a numerical example

Binomial betting - example: 50% chance to get +120% and 50% chance to loose -70%

- ✓ Excess return is $\mu = 0.5\ (120\% - 70\%) = 25\%$
- ✓ Volatility is 95%, Sharpe ratio is 0.26

After N-betting rounds [Redner],

- ✓ Most probable/median/time-average growth is $P_{mp} = ((2.2)^{0.5}(0.3)^{0.5}))^N = \exp(N * 0.5\ln(0.66))$
- ✓ Ensemble-average growth is $P_{ens-av} = (0.5 * 2.2 + 0.5 * 0.3)^N = \exp(N * \ln(1.25))$

- ✓ Kelly growth is $g(f) = p\ln(1 + bf) + (1 - \mathrm{p})\ln(1 - af)$
- ✓ Optimal Kelly fraction is $f^* = \frac{pb-(1-p)a}{ab} = 29\%$ to bet in each round, $g^* = 3.6\%$
- ✓ The wealth growth is $P_{optimal} = \exp(N * g^*) = \exp(N * 0.5\ \ln(1.074))$

- ➢ Kelly growth moves away exponentially from the median $P_{optimal} \gg P_{mp}$
- ➢ Kelly growth cannot reach the inaccessible ensemble-average $P_{optimal} \ll P_{ens-av}$
- ➢ Stochastic calculus is time-series average, not ensemble-average
- ➢ Leveraged betting, since f^* can be larger than 1

Kelly criterion (3) – foundations and applications

- Good properties of Kelly strategy
  - ✓ Maximizes geometric growth
  - ✓ Reaching a preassigned goal faster than any other strategy
  - ✓ Bettor has an optimal myopic strategy

- Bad properties of Kelly strategy
  - ✓ Large drawdowns are possible for the fractional-wealth multiplicative betting
  - ✓ “negative convexity” – the bettor is behind for exactly equal number of wins and losses

- The real life application - Thorp[2017] “A Man for All Markets”
  - ✓ Options trading (ahead of Black-Scholes-Merton), Market neutral strategy, Statistical arbitrage
    - Risk management of macro uncertainty
  - ✓ Blackjack and roulette
    - Complete versus incomplete information games (examples: blackjack versus poker)

- Foundations
  - ✓ For multi-period repeated independent trials, geometric growth rate becomes equivalent to log-utility, therefore many general results of utility theory appear applicable
  - ✓ **Majority of modern economists (after J. Bernoulli and P. Samuelson) prefer general utility theory on conceptual grounds**
  - ✓ The difference between time-series average and ensemble average [Peters]

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Kelly criterion with tail risk/opportunities – a numerical example

Example: 50% chance to get +120% and 50% chance to loose -70%

- ✓ Excess return is $\mu = 0.5\,(120\% - 70\%) = 25\%$
- ✓ Volatility is 95%, Sharpe ratio is 0.26
- ✓ Optimal Kelly fraction is $f^* = 29.7\%$ to bet in each round, $g^* = 3.6\%$

Tail risk of 3% to loose -90% which is -2.7% off excess return

- ✓ Optimal Kelly fraction is $f^* = 25.1\%$ to bet in each round, $g^* = 2.63\%$

Tail opportunity of 1% to gain 270% which is +2.7% additional excess return

- ✓ Optimal Kelly fraction is $f^* = 31.7\%$ to bet in each round, $g^* = 4.16\%$

BOTH of the above - tail risk and opportunity

- ✓ Optimal Kelly fraction is $f^* = 27\%$ to bet in each round, $g^* = 3.13\%$

- ➢ **Non-linear and sensitive effects of tail risks and opportunities**

Kelly criterion and tail-risk uncertainty (1)

Risk is the known unknown
Uncertainty is the unknown unknown

- Uncertainty and Risk due to different reasons
  - A) genuine unpredictability, i.e. tail-risk, natural disasters, etc.
    Actions: reduce positions and diversify
  - B) **The true source of uncertainty lies in the intentions of others**, i.e. Central Banks, other investors, principals, etc.
    Actions: exploit behavioural biases and apply game strategies

- Kelly leverage – concentrated portfolios (Thorp, Buffett, Paul Tudor Jones, etc.)
  - ✓ Leverage aversion and the portfolio theory [Asness 2012]

- Skew and convexity effects
  - ✓ *"Wrong-way risk" (leverage aversion) - drawdown and (too high) leverage*
  - ✓ *"Right-way risk" (risk reduction) between future risk-premium and inverse variance*

- Tail risk and Uncertainty
  - ✓ Dark matter of finance – probability of catastrophic event&impact [Ross 2011]
  - ✓ Connection between Drawdown aversion and Leverage aversion [Turlakov 2017]
  - ✓ Cycles and value of tail risk - extracting information content [Bhansali]
  - ✓ Pandora's Risk – the major role of uncertainty in financial markets – K. Osband

Kelly criterion and tail-risk uncertainty (2)

Uncertainty is the impossibility to predict and quantify

- **Leverage and Uncertainty [Turlakov, 2017]** - Kelly criterion with tail risk uncertainty
  - ✓ RISK is ATM volatility, UNCERTAINTY is tails/skews
  - ✓ ETL and α are Expected Tail Loss and the probability weight correspondingly

Geometric growth rate is suppressed due to tail-risk

$$g(f_1^*) \approx \frac{\mu^2}{2\sigma^2}(1-\alpha)\frac{\left(1-\frac{\alpha ETL}{\mu(1-\alpha)}\right)^2}{\left(1+\frac{\alpha ETL^2}{\sigma^2(1-\alpha)}\right)}$$

Kelly Parity – multi-asset portfolio based on Kelly strategy

- **Kelly Parity is more general and inclusive of Markowitz and Risk Parity**
  - ✓ If total leverage is one (and away from the asset-allocation boundaries), Kelly Parity becomes Mean-variance Optimization (Markowitz)
  - ✓ If Sharpe ratios are the same for all assets, Kelly Parity becomes Risk Parity (Dalio)

- **Kelly-Thorp portfolio theory is superior**
  - ✓ [Thorp 1969] showed explicit example. Multi-period problem points to Kelly-Thorp strategy
  - ✓ Leverage on the efficient frontier can be determined only beyond Markowitz theory
  - ✓ Ensemble-average and utility theory are linear leverage

- Kelly-Thorp portfolio – possible to interpolate between Risk Parity and Tail-Risk Parity

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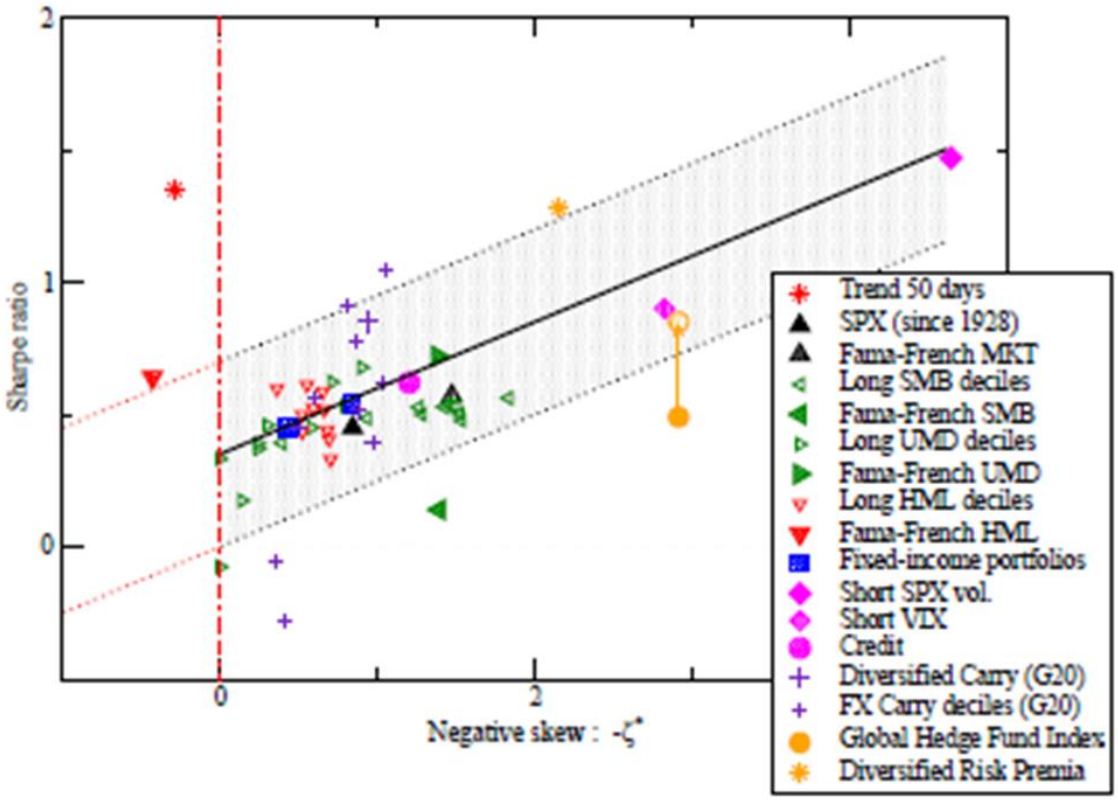
General observations about trend/momentum

"Now, on to the myth busting. … Myth 10: there is no theory behind momentum" – C. Asness

- Momentum is "premier anomaly" [Lemperiere]
  - ✓ Behavioural and risk-based reasons and the transformation of information
  - ✓ Markets is the mechanism of the transformation of uncertainty into risk
  - ✓ Positive Convexity (and skew?) – "small losses and few large wins" – Bouchaud et al., R. Martin



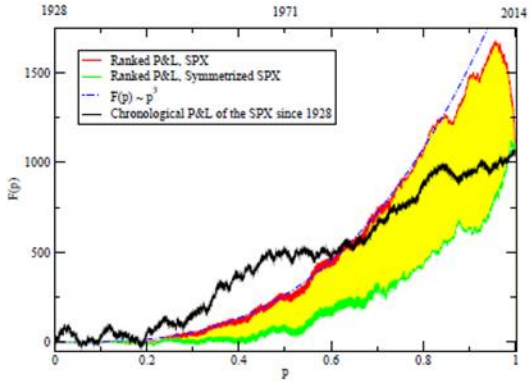
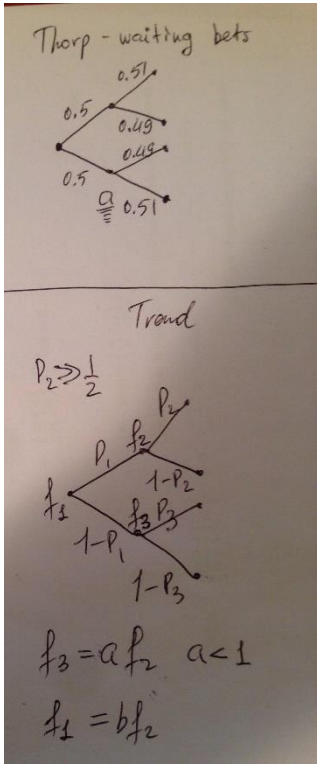


Figure 1: The ranked amplitude P&L representation: plot of the cumulated daily P&L $F(p)$ for the SPX (at constant risk) since 1928, as a function of the normalized rank of the *amplitude* of the returns, $p = k/N$ (in red). The standard, chrono-

Kelly-Thorp criterion for trend (1)

- Two-period Kelly strategy
  - ✓ Waiting bets - scaling of bets on favourable and unfavourable tree [Thorp]
  - ✓ Unfavourable effect of unfavourable tree for $0 < a < 1$

$$f_a = 2\frac{(P_u - 0.5)}{\sigma}\frac{1-a}{1+a^2}$$



  - ✓ **Stay in the game – have a chance to make the trend your friend.** Growth can be enhanced up to a factor of 2.5 compared to simple final-state optimisation. Scaling up on the trend tree and down off the favourable tree

$P_1 \sim 1, P_2 \sim 1, b \sim 1$ on the trend tree
$P_3 \sim 0, a \sim 0$ unfavourable tree

Kelly-Thorp criterion for trend (2)

- Proebsting's paradox
  - ✓ Change of odds after making a first move and before the new favourable outcome
  - ✓ **Additional information makes MtM profit/loss for the bettor outside of his control**
  - ✓ A possibility of misspecification of Kelly parameters requires lower Kelly fraction, similar result to negative tail risk
- Open questions
  - ✓ Autocorrelation is useful, yet risky, information and therefore should be exploitable
  - ✓ Conditional probability and path dependence break the simplicity of utility theory
  - ✓ **Opportunity to exploit the right fat tail**
  - ✓ *how to distinguish risk premium and autocorrelation in historical time-series?*

Applications of the game theory to liquid financial markets

- General game theory concepts, economists and markets
  - ✓ Theoretical game theory and utility functions – Nash, von Neumann, Aumann, …
  - ✓ **Experimental game theory** – Vernon Smith
  - ✓ **Behavioural finance**, uncertainty, prospect theory – Kahneman and Tversky
  - ✓ **Asymmetric information** – Ackerlof, Spence, Stiglitz

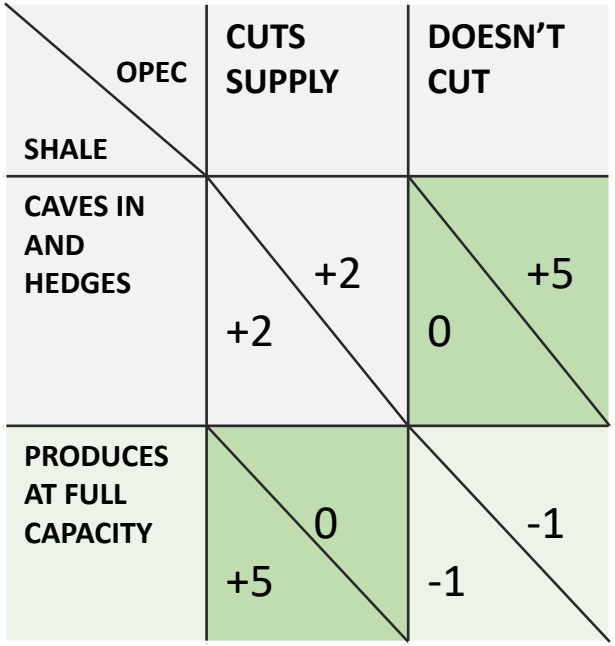
- **Practical applications and "one-shot" situational analysis**
  - ✓ Information Theory, Game Theory, and Common Knowledge – Ben Hunt
  - ✓ Markets and Poker – one-shot and multi-shot strategies - Aaron Brown

- **Practical application to the game interactions of strategies**
  - ✓ The strategies/factors are clearly defined and declared
  - ✓ Relevant games – Coordination and anti-Coordination Games with Common Knowledge
  - ✓ Iterated games – Correlated Equilibria and Multiple-shot interactions

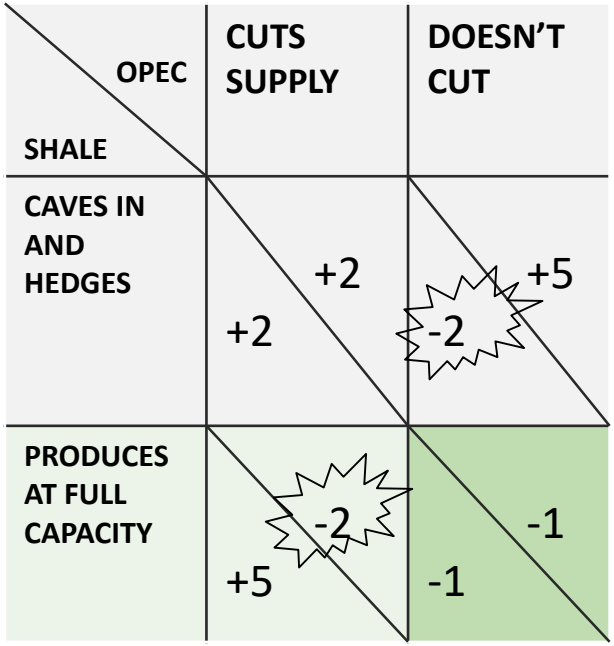
Example of game theory analysis – Shale vs OPEC

- Payoff diagrams – dynamic multi-period competitive game
  - ✓ Tactical (Chicken Game) and strategic (Prisoner's Dilemma?) payoffs are different
  - ✓ 2015 – Prisoner's Dilemma, from end of 2016 – Chicken game with Nash (left bottom corner)
  - ✓ 2nd half of 2017&2018 – right upper corner of Chicken Game or back to Prisoner's Dilemma?

Chicken (anti-cooperation) Game



Prisoner's **Dilemma**



- ✓ Correlated "equilibrium"/information – oil market price (determined by macro conditions)

Future outcomes – competition or cooperation? Mixed or correlated probabilistic strategies?

Hierarchy of the relevant games from simple to complex

Just play. Have fun. Enjoy the game.
-Michael Jordan-

- Kelly game – known biased coin betting
  - ✓ Permanent information edge and fixed excess return
  - ✓ Exponential wealth growth from “infinite market”
  - ✓ The solved problem - **optimal long-run** betting [Kelly]

- Bell-Cover game – two-player randomized-wealth biased-coin game
  - ✓ Higher wealth wins but each player’s initial wealth is randomized. Common signal/market
  - ✓ Similar to simple Matching-pennies Game, where randomization/noise appears naturally
  - ✓ Optimal strategy using Kelly criterion is known. **Short-run strategy**
  - ✓ Typical game-theoretic stalemate – players have equivalent strategies and no excess return
  - ✓ Exploitable edge if one player is not playing optimally

- Iterated Coordination and anti-Coordination Games with Common Knowledge/Market – proxy to real markets
  - ✓ Different one-shot and multiple-shot strategy?
    - Example: Prisoner’s Dilemma – confrontational short-run and cooperative long-run
  - ✓ Nash pure, mixed strategy or “correlated equilibrium” [Aumann]?
  - ✓ Risky dynamic edge in the evolving game with shifting equilibria?

Game theory interactions between investment strategies

A thriving manager: "Who is losing money that I am going to make for you?"

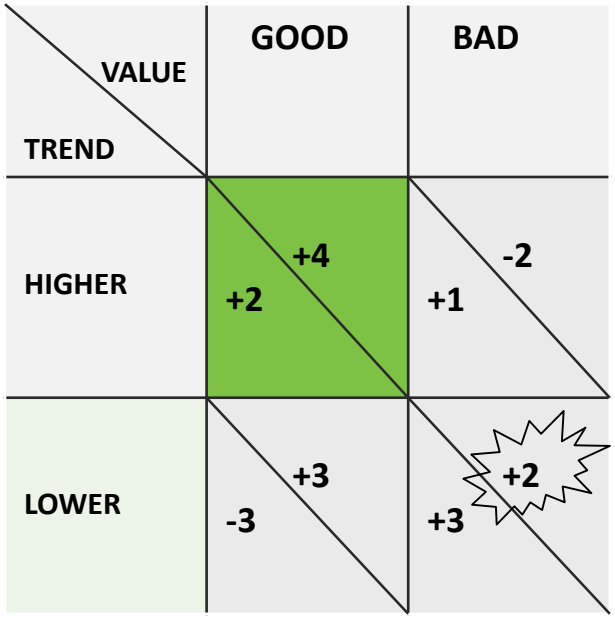
- ✓ Trend followers (winners) versus Return chasers (losers) [Haghani, 2016]
- ✓ Benchmarkers versus Momentum traders [Vayanos]

- ❑ Substantiating the proposal
  - ✓ **Stable and explicit decision-making process for active management**
  - ✓ Forward-looking meta-strategy – not equilibrium and not fully pre-determined by backtest biases
  - ✓ In markets, people and strategies are the most rational than in any other activities
  - ✓ **Behavioural biases can be included into the framework**
  - ✓ **Not only price data but importantly, volume and positioning data are helpful**
  - ✓ Suitable for global macro - collective/macro news
  - ✓ Equities – negative sum game in short-run (taxes and fees) and positive sum in long-run
  - ✓ Zero-sum game for FX, rates and commodities

- ❑ Limitations
  - ✓ Distribution of parameters/views within a single strategy
  - ✓ Difficult to calibrate, based on partial information, plenty of subjectivity and noise
  - ✓ Possibly not simultaneous decisions – leader-follower relationship
  - ✓ "More is different" – collective networks - abrupt (crash) and slow (trend) "phase transitions"

Toy example - value versus absolute momentum

- Payoff diagram for certain stage of US equity market
  - ✓ Coordination Game with mixed strategy or correlated equilibrium? High and low transition barriers between different states and types of games?
  - ✓ More realistic and insightful 3-states game matrix - Value (Good, return-chaser, Bad) and Trend (Higher, Mean-Reversion, Lower)
  - ✓ **Common Knowledge/"correlated equilibrium" – CBs liquidity, growth, inflation**



Summary

Exploit and explore the information and the games **in practice** and in theory

- Kelly criterion is fragile under tail risk
  - ✓ Scale down leverage and neutralise tail risk
- Kelly Parity encompasses Markowitz Mean Variance, Risk Parity and Tail-Risk Parity
  - ✓ In the limiting cases, Kelly Parity becomes equivalent to other famous portfolio theories
- Kelly-Thorp criterion for absolute momentum (new insights)
  - ✓ Trend is your friend, leverage it well
- Game theory interactions between dual strategies (work in progress)
  - ✓ Forward-looking asset-allocation decision making process

Disclaimer

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