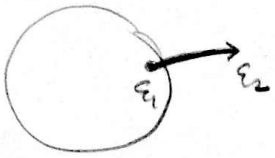
4th order expansion using TLS

$$\sum_{\omega_n} \frac{1}{(\omega_n-\varepsilon_1)^2(\omega_n-\varepsilon_2)^2}$$

$$= \int \frac{n_F(z)}{(z-\varepsilon_1)^2(z-\varepsilon_2)^2} = \left[\frac{n(z)}{(z-\varepsilon_2)^2}\right]'_{\varepsilon_1} + \left[\frac{n(z)}{(z-\varepsilon_1)^2}\right]'_{\varepsilon_2}$$

$$= 2\frac{n(\varepsilon_1)-n(\varepsilon_2)}{(\varepsilon_1-\varepsilon_2)^3} + \frac{n'(\varepsilon_1)+n'(\varepsilon_2)}{(\varepsilon_1-\varepsilon_2)^2}$$



$$E = E_1 n_F(E_1) + E_2 n_F(E_2)$$

2nd order:

$$E^{(2)} = \left(\varepsilon_1 + \frac{\Delta^2}{\varepsilon_1-\varepsilon_2}\right) n_F\left(\varepsilon_1 + \frac{\Delta^2}{\varepsilon_1-\varepsilon_2}\right) + \left(\varepsilon_2 - \frac{\Delta^2}{\varepsilon_1-\varepsilon_2}\right) n_F\left(\varepsilon_2 - \frac{\Delta^2}{\varepsilon_1-\varepsilon_2}\right)$$

$$= E_0 + \frac{\Delta^2}{\varepsilon_1-\varepsilon_2}\left(n_F(\varepsilon_1)-n_F(\varepsilon_2)\right) + \varepsilon_1 n_F'(\varepsilon_1)\frac{\Delta^2}{\varepsilon_1-\varepsilon_2} - \varepsilon_2 n_F'(\varepsilon_2)\frac{\Delta^2}{\varepsilon_1-\varepsilon_2}$$

$\to \not{\varnothing}$ BC $\int d\varepsilon_1 \varepsilon_1 \delta(\varepsilon_1) \to 0$

$$\Delta E^{(4)} = -\frac{2\Delta^4}{(\varepsilon_1-\varepsilon_2)^3} n_F(\varepsilon_1) + \frac{2\Delta^4}{(\varepsilon_1-\varepsilon_2)^3} n_F(\varepsilon_2)$$

$$+ \frac{\Delta^4}{(\varepsilon_1-\varepsilon_2)^2}\left(n_F'(\varepsilon_1)+n_F'(\varepsilon_2)\right)$$

$$+ \varepsilon_1 \frac{1}{2}\frac{\Delta^4}{(\varepsilon_1-\varepsilon_2)^2} n_F''(\varepsilon_1) + \varepsilon_2 \frac{1}{2}\frac{\Delta^4}{(\varepsilon_1-\varepsilon_2)^2} n_F''(\varepsilon_2)$$

$\to \not{\varnothing}$ BC $\int d\varepsilon_1 \varepsilon_1 \delta'(\varepsilon_1) \to 0$

$$E_{1,2} = \frac{\varepsilon_1+\varepsilon_2}{2} \pm \sqrt{\frac{(\varepsilon_1-\varepsilon_2)^2}{4} + \Delta^2}$$

$$= \frac{\varepsilon_1+\varepsilon_2}{2} \pm \frac{\varepsilon_1-\varepsilon_2}{2}\left(1+\frac{2\Delta^2}{(\varepsilon_1-\varepsilon_2)^2}\right) - \frac{1}{4}\left(\frac{4\Delta^2}{(\varepsilon_1-\varepsilon_2)^2}\right)^2$$

$$= \begin{cases} \varepsilon_1 + \dfrac{\Delta^2}{\varepsilon_1-\varepsilon_2} - \dfrac{2\Delta^4}{(\varepsilon_1-\varepsilon_2)^3} \\ \varepsilon_2 - \dfrac{\Delta^2}{\varepsilon_1-\varepsilon_2} + \dfrac{2\Delta^4}{(\varepsilon_1-\varepsilon_2)^3} \end{cases}$$