

On the Ultrasonic Attenuation in Glasses at Low Temperatures

J. Jäckle

Département de Physique Théorique, Université de Genève,
Genève, Switzerland

Received August 4, 1972

The ultrasonic attenuation in glasses at low temperatures is calculated using the model proposed by Anderson, Halperin, and Varma. It is found that the resonant absorption of sound by the localized two-level systems saturates and can be observed only if the sound amplitude is extremely small. A second contribution to the sound absorption is derived which arises from the relaxation of the localized excitations and does not saturate. Qualitative agreement with recent measurements of the ultrasonic attenuation in fused silica is obtained.

1. Introduction

The recent observation¹ of an anomalous temperature dependence of the specific heat and the thermal conductivity of glasses at temperatures below about 1 K has stimulated theoretical investigation into the thermal and acoustical properties of amorphous insulators. Fulde and Wagner (FW)² proposed a theory which is based on the assumption that structural relaxation leads to a diffusive mode which is especially important at short wavelengths. A more explicit model has been put forward by Anderson, Halperin, and Varma (AHV)³ who assume that a certain number of atoms have two equilibrium positions in an asymmetric double-well potential. Transitions between the two positions are possible by tunneling, whereby a resonant phonon is absorbed or emitted. A similar model has been developed by Phillips⁴.

Both theories predict that the ultrasonic attenuation should increase with decreasing temperature below about 1 K. According to AHV, this anomalous temperature dependence results from the resonant absorption of sound quanta by localized two-level systems. Since the absorption by two-level systems saturates for high power input, the contribution of resonant absorption to the ultrasonic attenuation saturates for large sound amplitudes. This mechanism was already proposed by Heinicke,

1 Zeller, R. C., Pohl, R. O.: Phys. Rev. B **4**, 2029 (1971).

2 Fulde, P., Wagner, H.: Phys. Rev. Letters **27**, 1280 (1971).

3 Anderson, P. W., Halperin, B. I., Varma, C. M.: Phil. Mag. **25**, 1 (1972).

4 Phillips, W. A.: J. Low Temp. Phys. **7**, 351 (1972).

Winterling, and Dransfeld⁵ in order to explain the discrepancy between thermal and coherent phonon mean free paths. We estimate the value of the power input at which saturation effects become important. We find that for typical experimental values of the power which is fed into the ultrasonic wave the resonant absorption of sound is indeed saturated. In this case the linear attenuation coefficient which results from resonant absorption is strongly reduced and cannot be observed experimentally.

For a comparison of the two theories it is of interest to see how structural relaxation, which is the basic concept of FW, is described by the model of AHV. Since the two equilibrium positions of a particular atom correspond to different atomic configurations, the tunneling transition can be considered as a mechanism of structural relaxation. The role of the phonons in this process is to transport energy from "hot atoms" to "cold atoms". We derive the sound absorption which results from this relaxation process. Since in the relaxation process the ultrasonic wave interacts with all two-level systems which have an excitation energy in the thermal energy range the relaxation absorption does not saturate. We compare the relaxation contribution with recent experimental results⁶ for the high-frequency ultrasonic attenuation in fused silica at temperatures down to 1.1 K. Qualitative agreement between theory and experiment is obtained.

2. The Model

We first describe the model proposed by AHV in order to explain the observed anomalies of the specific heat and the thermal conductivity. Since the specific heat was found¹ to vary linearly with temperature the first objective of such a model must be to introduce excitations which have a constant density of states in the energy range of interest. The first hypothesis of AHV is that in a glass a certain number of atoms (or groups of atoms) exist, which have two equilibrium positions corresponding to the minima of asymmetric double-well potentials (Fig. 1). The motion of such atoms can be approximately described as an oscillation around either of the two potential minima. At low temperature the localized oscillators are in their ground states. We denote the ground states corresponding to the two equilibrium positions by $|\alpha\rangle$ and $|\beta\rangle$. These states are not exact eigenstates because of the tunneling through

5 Heinicke, W., Winterling, G., Dransfeld, K.: Proceedings of the Second Intern. Conf. on Light Scattering in Solids, ed. M. Balkanski, p. 463. Paris: Flammarion 1971.

6 Arnold, W., Baumann, J., Berberich, P., Hunklinger, S., Leiderer, P., Nava, R., Dransfeld, K.: Proceedings of the Intern. Conf. on Phonon Scattering in Solids, ed. H. J. Albany, p. 359, Paris: La Documentation Française 1972.

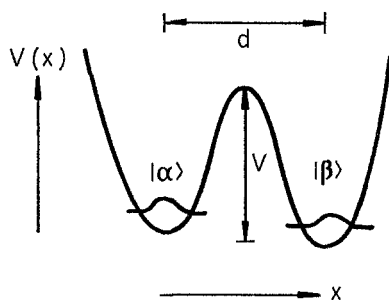


Fig. 1. Asymmetric double-well potential with localized oscillator ground states

the potential barrier from one minimum to the other. The properties of a quantum-mechanical two-state system are determined by a hamiltonian matrix

$$H_0 = \frac{1}{2} \begin{pmatrix} \varepsilon & \Delta \\ \Delta & -\varepsilon \end{pmatrix}. \quad (1)$$

ε is equal to the energy difference between the localized ground states which is the sum of the energy difference between the two potential minima and the difference in zero-point energy of the two oscillators. The off-diagonal matrix element Δ is due to the overlap of the wavefunctions of the localized oscillators which gives rise to tunneling from one position to the other. Δ can be written as

$$\Delta = \hbar \omega_0 e^{-\lambda}, \quad (2)$$

where $\hbar \omega_0$ is of the order of the zero-point energy for the motion of an atom around one potential minimum (which may be of the order of 10^{-2} eV), and λ is roughly proportional to the square-root of the height V of the potential barrier and to the distance d between the two potential minima (see Fig. 1). The eigenstates $|1\rangle$ and $|2\rangle$ of H_0 have energies

$$E_{1,2} = \pm \frac{1}{2} E \quad \text{with} \quad E = \sqrt{\varepsilon^2 + \Delta^2}. \quad (3)$$

They represent a two-level system with level spacing equal to E . We are interested in low temperatures where only those two-level systems contribute to the thermal properties which have a separation in energy $E \ll \hbar \omega_0$. Therefore, we have to consider only double-well potentials with values of Δ which are much smaller than $\hbar \omega_0$. As a consequence, the exact value or the distribution of the values of ω_0 is not relevant for our results.

In order to explain the linear temperature dependence of the specific heat the two-level systems must have a uniform distribution of the excitation energy E . This requirement is met by AHV by assuming that the

parameters ε and λ are uniformly distributed in the range important at low temperature. With this assumption we have for the probability distribution $P(\varepsilon, \lambda)$ (for 1 cm^3 of the substance)

$$P(\varepsilon, \lambda) = \bar{P}, \tag{4}$$

where \bar{P} is a constant. From the measured specific heat¹ we derive a value of $4.35 \cdot 10^{31} \text{ erg}^{-1} \text{ cm}^{-3}$ for \bar{P} .

In order that the ensemble of localized two-level systems can respond to an external perturbation a mechanism must exist through which transitions between the eigenstates $|1\rangle$ and $|2\rangle$ become possible. Such a mechanism is provided by the interaction with long-wavelength phonons. The coupling between the localized two-level systems and the phonons derives from the deformation of the double-well potentials which is caused by an elastic wave⁷. The deformation adds a perturbation H_1 to the hamiltonian for a two-state system. We neglect for simplicity a possible variation of the off-diagonal matrix element and write

$$H_1 = \frac{1}{2} \begin{pmatrix} \delta\varepsilon & 0 \\ 0 & -\delta\varepsilon \end{pmatrix}. \tag{5}$$

Expressing $\delta\varepsilon$ in terms of the elastic strains by means of a deformation potential parameter B (which we assume to be the same for longitudinal and transverse waves), and the elastic strains in terms of phonon creation and annihilation operators⁸ we obtain the matrix element for absorption or emission of a phonon of momentum \mathbf{k} and polarization α

$$\langle \mathbf{k}, \alpha; 2 | H_1 | 1 \rangle = \left(\frac{k}{2\rho c_\alpha} \right)^{\frac{1}{2}} B \frac{\Delta}{E}. \tag{6}$$

Here ρ is the density of the glass and c_α stands for the (longitudinal or transverse) sound velocity. Given the transition matrix element, one can calculate the rate τ_j^{-1} at which a particular two-level system j in the upper state emits a phonon thereby returning to the lower state, and the rate $(\tau_{\mathbf{k}, \alpha})^{-1}$ at which a phonon with momentum \mathbf{k} and of polarization α is absorbed by one of the localized modes with which it is at resonance. Using the golden rule we obtain

$$\tau_j^{-1} = \left(\frac{1}{c_l^5} + \frac{2}{c_t^5} \right) \frac{B^2 \Delta_j^2 E_j}{2\pi\rho\hbar^4} \text{ctgh} \left(\frac{\beta E_j}{2} \right) \tag{7}$$

and

$$(\tau_{\mathbf{k}, \alpha})^{-1} = \frac{\pi B^2 \bar{P} \omega}{\rho c_\alpha^2} \text{tgh} \left(\frac{\beta \hbar \omega}{2} \right), \quad \omega = c_\alpha \cdot k, \beta^{-1} = k_B T. \tag{8}$$

7 Süßmann, J. A.: Phys. of Condensed Matter **2**, 146 (1964); — J. Phys. Chem. Solids **28**, 1643 (1967).

8 Kittel, C.: Quantum theory of solids, chap. 2. New York: J. Wiley 1963.

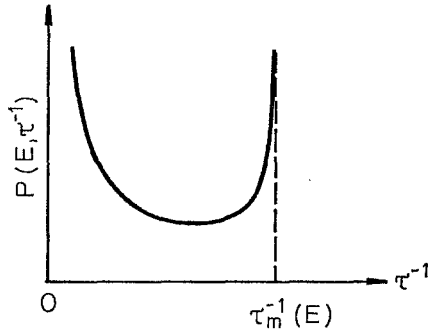


Fig. 2. Distribution of relaxation rates τ^{-1} of two-level systems

The factor $(c_l^{-5} + 2c_t^{-5})$ appears in the expression for the relaxation rate τ_j^{-1} because the emitted phonon can belong to either the longitudinal or one of the two transverse phonon branches. It follows from Eq. (7) that the maximum relaxation rate $\tau_m^{-1}(E)$ of a two-level system with excitation energy $E = \sqrt{\varepsilon^2 + \Delta^2}$ is obtained for $\Delta = E$ and $\varepsilon = 0$. For the values of the parameters c_l , c_t , and ρ of quartz glass

$$c_l = 5.8 \cdot 10^5 \text{ cm sec}^{-1}, \quad c_t = 3.75 \cdot 10^5 \text{ cm sec}^{-1}, \quad \rho = 2.2 \text{ g cm}^{-3},$$

and the parameter B which we fit to the ultrasonic attenuation (Chapter 4) we get the numerical value

$$\tau_m^{-1}(E) = 1.95 \cdot 10^8 (E/k_B)^3 \text{ctgh}(\beta E/2) [\text{sec}^{-1}]. \quad (9)$$

The probability distribution $P(E, \tau^{-1})$ for finding a two-level system with excitation energy E and relaxation rate τ^{-1} turns out to be

$$P(E, \tau^{-1}) = \frac{\bar{P}}{2} \frac{\tau}{(1 - \tau_m(E)/\tau)^{1/2}} \quad \text{for } \tau > \tau_m(E). \quad (10)$$

The remarkable feature is the strong increase of the distribution function $P(E, \tau^{-1})$ at very long relaxation times (Fig. 2). The atoms with very long relaxation times are those with a high potential barrier separating the two potential minima. As AHV have pointed out, this distribution of relaxation times leads to a logarithmic dependence of the measured specific heat on the time the sample is given to respond to the external heat supply in the experiment. A change of this response time from 1 sec to 1 μsec should reduce the specific heat by a factor of 3.

The phonon relaxation rate $\tau_{\vec{k}, \alpha}^{-1}$ depends on the temperature through the population factor

$$\text{tgh}\left(\frac{\beta E}{2}\right) = 1 - 2n_0(E), \quad n_0(E) = [\exp(\beta E) + 1]^{-1}.$$

(n_0 is the distribution function for the two-level systems in equilibrium.) With increasing temperature the resonant absorption decreases because of the equalization of the occupation of the two levels.

With the result (8) for the phonon absorption rate, one can explain³ the measured thermal conductivity¹ κ and predict the ultrasonic attenuation at low temperature. Assuming that heat is transported only by phonons, which are scattered by the localized modes, one obtains a T^2 -variation of κ which agrees roughly with the measured temperature dependence. Comparison with the measured absolute value of κ yields an estimate of the deformation potential B of the order of 1 eV. For ultrasonic phonons ($\hbar\omega \ll k_B T$) AHV predict a variation of the attenuation with frequency and temperature as ω^2/T . The attenuation should decrease as $1/T$ with increasing temperature since the resonant scatterers are bleached out with increasing population of the upper energy level.

3. Saturation of the Resonant Absorption

We mentioned already that the resonant absorption of phonons by the localized two-state systems depends on the population of the energy levels. If an ultrasonic wave has a sufficiently small amplitude it does not affect the occupation numbers of the energy levels (averaged over one period of the sound wave). For larger sound amplitudes, however, the influence of the impressed sound wave on the occupation numbers may become important and lead to non-linear effects in the sound absorption. In fact, since the absorption of a sound quantum corresponds to the transition of an atom from the ground state to the excited state, it is clear that the population of the upper levels increases in proportion to the energy which is absorbed from the ultrasonic wave. If the amplitude of the sound wave is increased further, a situation is finally obtained in which the two energy levels are equally populated. In this situation the absorption of energy is completely determined by the rate of the recombination process in which the excited atoms return to their ground states. The absorption of energy has reached its saturation value which is independent of the sound amplitude.

We can calculate this saturation absorption since we know the relaxation time for phonon emission of an excited two-level system. The recombination rate \dot{n}_j of the two-level system j is given by

$$-\dot{n}_j = \frac{n_j(1 + N_0(E_j)) - (1 - n_j)N_0(E_j)}{\tilde{\tau}_j}, \quad (11)$$

where $\tilde{\tau}_j^{-1}$ is the relaxation rate (7) without the population factor:

$$\tilde{\tau}_j^{-1} \operatorname{ctgh} \left(\frac{\beta E_j}{2} \right) = \tau_j^{-1},$$

and $N_0(E)$ is the Bose function

$$N_0(E) = [\exp(\beta E) - 1]^{-1}.$$

The rate (11) attains its maximum value $\dot{n}_j^{(s)}$ for $n_j = 1/2$. Since in the stationary case the rates of recombination and excitation are equal, the saturation value of the energy absorption is given by

$$\dot{Q}_s = \hbar \omega \sum_{j, |E_j - \hbar \omega| < \Gamma} \dot{n}_j^{(s)}. \quad (12)$$

Here we sum over all two-level systems satisfying the resonance condition which requires that the excitation energy fits the ultrasonic frequency within the combined energy and frequency uncertainties (denoted by Γ). The sum can be evaluated using the distribution $P(E, \tau^{-1})$ of excitation energies and relaxation times given in Eq. (10):

$$\dot{Q}_s = \hbar \omega \frac{1}{2} 2\Gamma \int_0^\infty d(\tau^{-1}) \tilde{\tau}^{-1} P(\hbar \omega, \tau^{-1}).$$

The integral in this expression has the value $\bar{P} \cdot \tilde{\tau}_m^{-1}(\hbar \omega)$. The result for the maximum absorption (per unit volume and time) is

$$\dot{Q}_s = \left(\frac{1}{c_l^5} + \frac{2}{c_r^5} \right) \frac{B^2 \Gamma \bar{P} \omega^4}{2\pi \rho}. \quad (13)$$

For a frequency of 1 GHz and a relative energy uncertainty $\Gamma/\hbar \omega$ of $2^0/_{00}$ (which results from the finite relaxation time of the localized excitations and the uncertainty of the ultrasonic frequency⁹) this maximum absorption at 1 K is $8 \cdot 10^{-9}$ Watt cm^{-3} .

We estimate the acoustic power for which the resonant absorption begins to saturate by comparing the linear absorption rate with the maximum absorption \dot{Q}_s . For a given acoustic power j_e (per unit area) the linear absorption rate is equal to j_e/l , where the mean free path l is the product of the sound velocity and the phonon relaxation time (8). Equating the linear with the maximum absorption we find for the critical power input $j_e^{(s)}$ at which saturation effects become important

$$j_e^{(s)} = \dot{Q}_s \cdot l = \left(\frac{1}{c_l^5} + \frac{2}{c_r^5} \right) \frac{\Gamma c_a^3 \omega^3}{2\pi^2} \text{ctgh} \left(\frac{\beta \hbar \omega}{2} \right). \quad (14)$$

The value of the saturating power turns out to be extremely small. For a frequency of 1 GHz at 1 K, for instance, it is of the order of $4 \cdot 10^{-8}$ Watt cm^{-2} which is by about five orders of magnitude smaller than typical experimental values of the acoustic power input.

The ultrasonic attenuation of fused silica at low temperatures has recently been measured⁶ in an attempt to test the predictions of the theo-

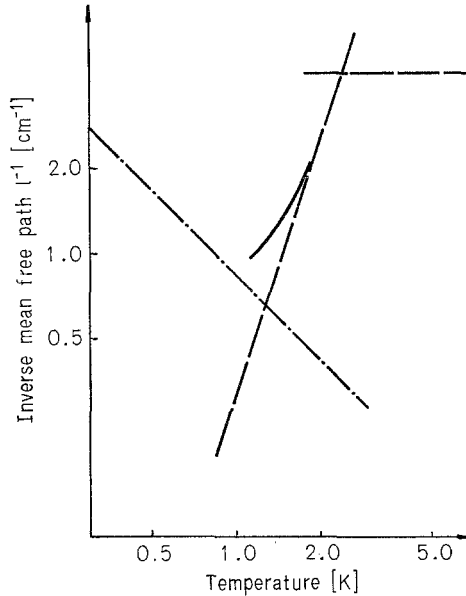


Fig. 3. Comparison of theory and experiment for the ultrasonic attenuation in fused silica at 2 GHz. Full line: experiment; dashed-dotted line: resonant absorption; dashed lines: relaxation absorption

ries of AHV and FW. Fig. 3 shows the experimental results for a frequency of 2 GHz and temperatures down to 1.1 K. The mean free path due to non-saturated resonant absorption is included for comparison. This has a value of 1.2 cm for 2 GHz at 1 K. Comparing the experimental with the theoretical curve one cannot rule out the possibility that the resonant absorption shows up at temperatures below 1.1 K. But the same authors have very recently extended the measurements to temperatures below 1 K (down to 0.5 K) and found no indication of the resonant absorption⁹. The preliminary results at 500 MHz show a continuous decrease of the attenuation down to the lowest measured temperature. This result is in keeping with our conclusion that the resonant absorption should not be observable in a standard ultrasonic attenuation measurement because of saturation effects. In order to avoid saturation one would have to reduce the acoustic power by several orders of magnitude. Only experiments with extremely weak sound pulses could provide a direct test of the AHV model*.

* *Note added in proof:* After this paper had been completed a strong amplitude dependence of the ultrasonic attenuation at very low acoustic power has indeed been observed by Hunklinger, Arnold, Stein, Nava, and Dransfeld (to be publ. in Physics Letters).

⁹ Private communication by the authors of Ref. ⁶.

In the next chapter we consider the contribution to the ultrasonic attenuation which results from the relaxation of the tunneling states. This part of the absorption does not saturate. Some of the experimental results of Ref. ⁶ can be explained by this absorption mechanism.

4. Sound Absorption due to Relaxation

In the ensemble of two-level systems thermal equilibrium is established via transitions which involve the emission and absorption of phonons. The relaxation to equilibrium is mediated by the phonons which are exchanged between the localized two-level systems. In simple physical terms, the relaxation process can be explained as a transport of energy by phonons from "hot atoms" to "cold atoms". In the relaxation process energy is absorbed from the external perturbation and fed into the two-level systems and the phonon system. In the case where the perturbation is caused by a coherent sound wave the relaxation gives rise to sound absorption. We calculate now this contribution to the sound absorption which follows without any additional assumptions from the model of AHV.

We describe the ensemble of two-level systems and the phonon system by their distribution functions n_j and $N_{k,\alpha}$. n_j gives the time-dependent probability for finding the two-level system j in the excited state, $N_{k,\alpha}$ is the usual space- and time-dependent phonon distribution function. The variation of the distribution functions is determined by two coupled Boltzmann equations which contain the perturbation caused by the sound wave as a driving force. We separate the oscillating parts δn and δN of the distribution functions from the equilibrium distributions n_0 and N_0 defined earlier by writing

$$n_j = n_0(E_j) + \delta n_j, \quad N_{k,\alpha} = N_0(\varepsilon_{k,\alpha}) + \delta N_{k,\alpha}. \quad (15)$$

Linearizing with respect to δn and δN we obtain the linearized Boltzmann equations

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \frac{1}{\tau_j} \right) \delta n_j + \frac{\partial n_0}{\partial E_j} \frac{\partial}{\partial t} \delta E_j \\ & = \text{tgh} \left(\frac{\beta E_j}{2} \right) \sum_{k,\alpha} V_{j,\alpha}^2 \delta(\varepsilon_{k,\alpha} - E_j) \delta N_{k,\alpha}, \end{aligned} \quad (16a)$$

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \mathbf{V}_k \varepsilon_{k,\alpha} \cdot \mathbf{V}_r + \frac{1}{\tau_{k,\alpha}} \right) \delta N_{k,\alpha} + \frac{\partial N_0}{\partial \varepsilon_{k,\alpha}} \frac{\partial}{\partial t} \delta \varepsilon_{k,\alpha} \\ & = \text{ctgh} \left(\frac{\beta \varepsilon_{k,\alpha}}{2} \right) \sum_j V_{j,\alpha}^2 \delta(E_j - \varepsilon_{k,\alpha}) \delta n_j, \end{aligned} \quad (16b)$$

where δE_j and $\delta \varepsilon_{\mathbf{k}, \alpha}$ denote, respectively, the variation of the excitation energy of two-level system j and of the energy of the phonon (\mathbf{k}, α) under the influence of the sound wave, and

$$V_{j, \alpha}^2 = \frac{\pi B_j^2 \Delta_j^2}{\rho c_\alpha^2 E_j} \quad (17)$$

is essentially the square of the transition matrix element (6). In general one must solve the coupled equations (16a, b) for both oscillating parts δn and δN of the distribution functions. At high frequencies, however, we may assume that the phonons remain in thermal equilibrium. Neglecting the phonon disturbance we get immediately from Eq. (16a)

$$\delta n_j = -\frac{\partial n_0}{\partial E_j} \frac{\delta E_j}{1 + i(\omega \tau_j)^{-1}}. \quad (18)$$

The sound absorption is calculated as the ratio between the energy dissipation and the energy current per unit volume. The energy fed into the ensemble of two-level systems per unit time is given by the time average

$$\dot{Q} = \sum_j \langle n_j(t) \delta \dot{E}_j(t) \rangle_t, \quad (19)$$

and the energy current of the sound wave is equal to

$$j_\varepsilon = \rho v_s^2 c_\alpha, \quad (20)$$

where v_s is the velocity amplitude of the wave. Inserting the solution (18) for δn_j into (19) we obtain for the mean free path l of an ultrasonic phonon

$$\frac{1}{l_\alpha(\omega)} = \frac{\dot{Q}}{j_\varepsilon} = \frac{1}{2\rho c_\alpha^3} \sum_j \left(-\frac{\partial n_0}{\partial E_j} \right) \frac{\omega^2 \tau_j}{1 + \omega^2 \tau_j^2} \left(\frac{\delta E_j}{v_s/c_\alpha} \right)^2. \quad (21)$$

The contribution of every two-level system j shows the typical relaxation behaviour¹⁰ with the corresponding relaxation time τ_j . Summing over all two-level systems corresponds to averaging over the relaxation times τ_j . It is convenient to evaluate the sum over the two-level systems by averaging over the potential parameters ε and λ according to the substitution

$$\sum_j \rightarrow \bar{P} \int d\varepsilon d\lambda.$$

10 Herzfeld, K. F., Litovitz, T. A.: Absorption and dispersion of ultrasonic waves. New York: Academic Press 1959.

The energy change δE_j derived from the perturbation hamiltonian (5) is

$$\delta E_j = 2B \frac{\varepsilon_j}{E_j} \frac{v_s}{c_\alpha}. \quad (22)$$

It is worth mentioning that the accepted order of magnitude of B (1 eV) corresponds to a Grüneisen parameter of order 10^4 since around 1 K the relevant energies E_j are of the order of 10^{-4} eV. The large value of the deformation potential is the reason why the relaxation mechanism leads to an observable attenuation at low temperatures. In the limiting cases of low frequency or high temperature and high frequency or low temperature we find for the inverse mean free path of longitudinal phonons

$$l_h^{-1} = \frac{\pi B^2 \bar{P} \omega}{4 \rho c_l^3} \quad (23)$$

for high temperature where $\omega \bar{\tau}_m \ll 1$, and

$$l_l^{-1} = \left(\frac{1}{c_l^5} + \frac{2}{c_t^5} \right) \frac{3 \zeta(3) B^4 \bar{P} (k_B T)^3}{2 \pi \rho^2 c_l^3 \hbar^4} \quad (24)$$

for low temperature where $\omega \bar{\tau}_m \gg 1$. Choosing a value of 1.3 eV for the deformation potential parameter B we get the numerical results

$$l_h^{-1} = 2.2 \cdot 10^{-9} \nu \text{ [cm}^{-1}\text{]} \quad (25)$$

and

$$l_l^{-1} = 0.31 T^3 \text{ [cm}^{-1}\text{]}. \quad (26)$$

At low temperature the absorption is independent of frequency as is characteristic for a relaxation mechanism. In this region the neglect of the disturbance of the phonon distribution can be justified if

$$\omega \tau_{ph} \gg 1,$$

where τ_{ph}^{-1} is the averaged resonant absorption rate of thermal phonons. At high temperature a quadratic frequency dependence of the absorption may be expected for a relaxation mechanism, but we find a linear frequency dependence. This unusual behaviour has its origin in the existence of two-level systems with very long relaxation times which according to AHV are characteristic of the glassy state.

Fig. 3 shows a comparison of the calculated relaxation absorption Eqs. (25) and (26) with the measured absorption⁶ in fused silica at 2 GHz. The low-temperature part of the relaxation absorption is in reasonable agreement with experiment. It is unclear whether the flat high-temperature part of the absorption curve can be observed since at higher temperatures a different relaxation mechanism becomes important which is

due to transitions of activated atoms over the top of the potential barrier. It is known that this mechanism leads to a peak in the sound absorption at about 40 K¹¹. At low temperatures the contribution from this mechanism is difficult to estimate since it depends strongly on the unknown cut-off at low energies in the distribution of the activation energies.

This work grew out of many stimulating discussions with Dr. S. Hunklinger. I am very indebted to him for sharing with me his knowledge of amorphous solids and keeping me informed on the process of the experimental work. I wish to thank also Dipl.-Phys. W. Arnold, P. Berberich, P. Leiderer, Professors K. Dransfeld, R. Nava, H. Wagner, and Dr. M. Papoular for helpful discussions and comments.

¹¹ Anderson, O. L., Bömmel, H. E.: J. Am. Ceram. Soc. **38**, 125 (1955).

Josef Jäckle
Fachbereich Physik
Universität Konstanz
D-7750 Konstanz
Federal Republic of Germany