od is the study of nonlinear effects: Nonlinear terms can actually be isolated and averaged separately. We have not yet systematically exploited this possibility. We have verified only that in the expansion of the drift velocity in terms of the applied force, only the odd-power terms contain systematic contributions to the statistical average, as is obvious on the grounds of symmetry. The even-power terms are present only in the mechanical response. As a consequence, since the first nonvanishing term beyond linearity is quadratic for the mechanical response and cubic for the statistical one, the latter must have a wider linearity range.

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# Observation of Akhiezer and Landau-Rumer Regimes in the Frequency Dependence of Shear-Wave Lattice Attenuation in CdS

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Brillouin-scattering measurements on acoustoelectrically amplified flux are used to determine the room-temperature frequency dependence of the lattice attenuation of ultrasonic shear waves in CdS up to a frequency of 7 GHz. The results show clearly a transition from the Akhiezer $f^2$ regime to the Landau-Rumer $f^1$ regime. The transition frequency provides an estimate of about 35 psec for the average lifetime of the thermal phonons involved in the ultrasonic attenuation process.

In this communication we report on Brillouin-scattering measurements of room-temperature lattice attenuation of slow shear waves propagating perpendicularly to the $c$ axis in CdS. By extending the measurements up to 7 GHz, we were able to observe a clear transition from a quadratic frequency dependence ($f^2$) of the lattice attenuation $\alpha_l$ at low frequencies to a linear dependence at higher frequencies. We belive that this corresponds to a transition from the Akhiezer regime[1] to the Landau-Rumer regime,[2] and as far as we know this is the first time that the two regimes have been observed (at a single temperature) in one and the same material. The transition frequency is of considerable interest since it provides an estimate for the average lifetime $\tau$ of the thermal phonons involved in the ultrasonic attenuation process.[3] The Akhiezer $f^2$ law is expected to hold when $f \ll 1/2\pi\tau$. For $f \gg 1/2\pi\tau$, on the other hand, the energy $hf$ of the ultrasonic phonons becomes large compared to the uncertainty $(h/2\pi\tau)$ in the energy of the thermal phonons with which the ultrasonic phonons interact. Selection rules resulting from energy and momentum conservation are then imposed on the phonon-phonon interaction, changing its nature and its frequency dependence. This is the range in which the lattice attenuation is expected[3] to increase linearly with frequency (the Landau-Rumer regime).

The frequency dependence of $\alpha_l$ has been measured in many materials[4,5] and an $f^2$ dependence, as well as other powers of $f$ ranging between 1 and 2, have been reported. As to shear waves in CdS, extensive data[6-10] are available below about 4 GHz, the latest and most accurate results[9,10] exhibiting clearly an Akhiezer $f^2$-law dependence. No measurements were reported so far at higher frequencies.

The measurements to be presented here were carried out on acoustoelectrically amplified flux[9,11] in highly homogeneous semiconducting CdS samples. The flux density and its spectral distribution were determined by Brillouin-scattering techniques, using the 5309-Å krypton-laser line. This particular wavelength was chosen in order to obtain maximum Brillouin-scattering efficiency.[12] The flux density at any phonon frequency was monitored by the intensity of the light scattered in the appropriate direction.[9] Extension of the frequency range was achieved by two means. First, low-resistivity samples (2–3 Ω cm) were used so as to amplify preferentially higher frequencies.[11] Second, the sample was immersed in a liquid of high index of refraction ($CS_2$), making it possible for the light to get into and out of the CdS sample at larger scattering angles so as to permit Brillouin probing at higher phonon frequencies.[13] The levels of amplified flux were kept sufficiently low throughout in order to ensure the applicability of the small-signal theory.[11] The Rayleigh-scattering signal was automatically subtracted out from the measurement and the net Brillouin signal was recovered by prolonged averaging. In this manner, probing of flux densities as low as 100 times the thermal-equilibrium density could be achieved without the need of spectroscopic techniques.[14]

The spatial distribution of acoustoelectrically amplified flux in a uniform sample has, under small signal conditions, the following form.[9,15] At any time $t$ following the onset of the amplifying current pulse, the flux density $\varphi$ varies essentially exponentially with the distance $x$ from the cathode, so long as $x \leq v_s t$ ($v_s$ being the sound velocity, $1.75\times10^5$ cm/sec for the slow shear waves in CdS). For $x \geq v_s t$, $\varphi$ is position independent and grows with time $t$ as[15]

$$\varphi(x,t)/\varphi_0=[1+(\alpha_0+\alpha_l)/\alpha_n]\exp(\alpha_n v_s t)-(\alpha_0+\alpha_l)/\alpha_n,\quad t\leq x/v_s, \tag{1}$$

where

$$\alpha_n \equiv \alpha_0\gamma-\alpha_l,\quad \gamma\equiv v_d/v_s-1. \tag{2}$$

Here $\varphi_0$ and $\varphi(x,t)$ represent, respectively, the thermal-equilibrium and amplified flux densities enclosed within a narrow cone in the direction of electron drift and in a narrow frequency bandwidth around the phonon frequency under study; $\alpha_0$ is the electronic amplification factor[11] and $\alpha_l$ the lattice attenuation at that frequency, both being independent of flux level under the small-signal conditions considered. The drift velocity $v_d$ and hence the parameter $\gamma$ defined in (2) can be obtained from the known electron mobility $\mu$ and the applied pulse amplitude. More accurately, $\gamma$ can be determined by means of the large-signal saturation current $I_m$, it being given by[16] $\gamma = I/I_m - 1$, where $I$ is the current through the filament.

Inspection of (1) shows that $\alpha_l$ and $\alpha_0$ can be determined from measurements of the flux density at any point in the plateau region ($x \geq v_s t$) as a function of $\gamma$ and $t$. Typical results of such measurements, taken at a frequency of 4.1 GHz, are shown in Fig. 1. The ordinate represents (on a log scale) the ratio of the Brillouin-scattered to incident light intensity $I/I_0$, a quantity that is directly proportional to the acoustic flux density $\varphi$. The lower curve (lower abscissa) represents the growth of $\varphi$ with time at a fixed value of $\gamma$, while the upper curve (upper abscissa) corresponds to the variation of $\varphi$ with $\gamma$ at a fixed time $t$. The

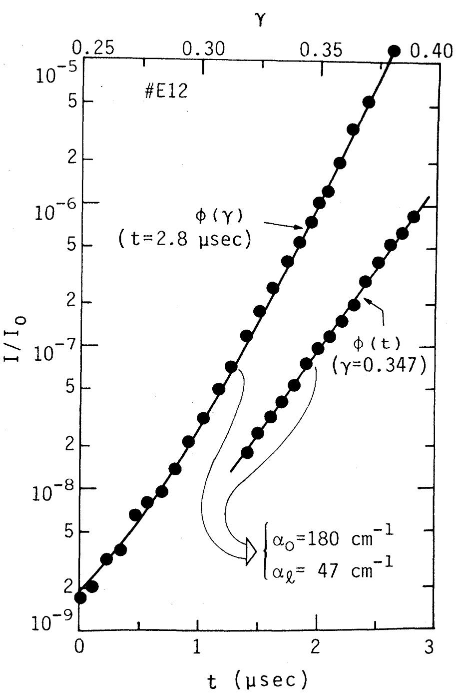


FIG. 1. Typical results ($f$ = 4.1 GHz) of growth of flux density $\varphi$ at distance $x$ = 5.5 mm from cathode with time $t$ ($< x/v_s$) following onset of amplifying current pulse and with $\gamma$. The flux density is monitored by the ratio of Brillouin-scattered to incident light intensity $I/I_0$. Curves represent Eq. (1) for values of $\alpha_0$ and $\alpha_l$ indicated. (Sample dimensions, $6.57\times1.825\times0.65$ mm$^3$; electron concentration, $n = 9.4\times10^{15}$ cm$^{-3}$; and mobility, $\mu$ = 260 cm$^2$/V sec.)

former curve is seen to be exponential, in accordance with (1), and the slope of the straight line yields directly the value of the net gain $\alpha_n$. The corresponding flux levels are sufficiently high to permit the neglect of the second term in (1) (which is independent of $t$) with respect to the first, exponential term. When the flux is measured as a function of $\gamma$, on the other hand, both this term as well as the pre-exponential term are functions of $\gamma$. Moreover, for low values of $\gamma$ for which $\alpha_n$ becomes small [see (2)], both terms are expected to give rise to a deviation from a strictly exponential dependence of $\varphi$ on $\gamma$ in (1). This deviation is clearly reflected in the $\varphi(\gamma)$ data shown in Fig. 1. The solid curves represent the best fit of (1) to the experimental points, obtained for the values of $\alpha_0$ and $\alpha_l$ indicated. The values determined in this manner are accurate to within several percent.

Results of the frequency dependence of the lattice attenuation $\alpha_l$, as obtained for two CdS samples from data of the type shown in Fig. 1, are plotted on a log-log scale in Fig. 2. Below about 4 GHz, an $f^2$ dependence is observed. The results in this spectral range are in good agreement in both slope and absolute magnitude with previously reported data (dot-dashed[9] and dashed[10] lines). Since the behavior in this range is well established, down to about 1 GHz, it has not been studied here in detail. Above about 4 GHz the quadratic dependence of $\alpha_l$ on $f$ is seen to give way gradually to a linear dependence characteristic of the Landau-Rumer regime.

Figure 3(a) displays the frequency dependence of the electronic amplification factor $\alpha_0$. The points represent results obtained (on the same two samples) by the procedure described above. The curve has been calculated from the Hutson-White small-signal theory[11] on the basis of the independently measured room-temperature val-

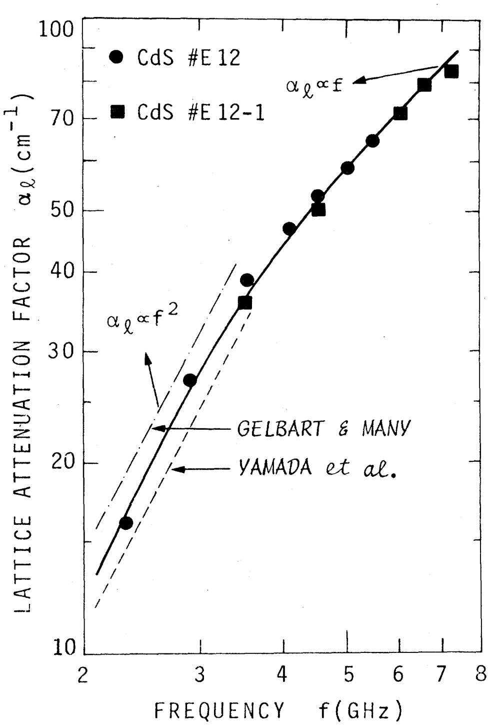


FIG. 2. Frequency dependence of lattice attenuation factor $\alpha_l$. Circles and squares refer to two different CdS samples having same values of $n$ and $\mu$ (see caption of Fig. 1). Curve passing through experimental points exhibits $f^2$ dependence at low frequencies and a linear dependence at higher frequencies. Also included are data obtained by other workers (Refs. 9 and 10) for the range 1–4 GHz, where $f^2$ dependence is also apparent.

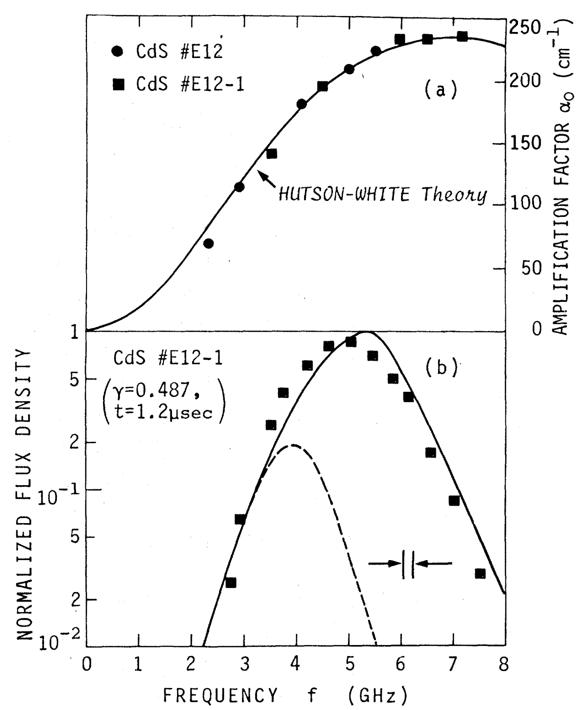


FIG. 3. (a) Frequency dependence of electronic amplification factor $\alpha_0$ for two samples of Fig. 2. Solid curve represents the Hutson-White theory (Ref. 11). (b) Frequency spectrum of amplified flux. Solid curve calculated on basis of the experimentally determined $\alpha_0(f)$ and $\alpha_l(f)$ values [Figs. 2 and 3(a)]. Dashed curve obtained if $f^2$ dependence of $\alpha_l$ at low frequencies is assumed to hold at high frequencies as well. Both curves normalized so as to coincide with experimental results at low frequencies.

ues of the electron concentration and mobility in the samples studied. The only other parameter entering the calculation is the electromechanical coupling constant $K^2$, for which an accurate value is available.[17] The agreement between theory and experiment is seen to be very good, a fact that reflects on the accuracy of the results obtained for both $\alpha_0$ and $\alpha_l$. Figure 3(b) provides a further independent check on the reliability of the $\alpha_l$ data presented, especially in the high-frequency range which is the range of most interest in this communication. Here the measured frequency spectrum of the amplified flux at one stage of its evolution (points) is compared with the spectrum calculated from (1) on the basis of the *experimentally* determined frequency dependence of $\alpha_0$ and $\alpha_l$ (solid curve). The calculated curve is seen to agree well with the data. It should be stressed that the shape of the theoretical curve depends strongly on both the magnitude and frequency dependence of $\alpha_l$. For example, if the Akhiezer $f^2$ dependence observed at low frequencies were to be extrapolated to higher frequencies (4–7 GHz), the calculated spectrum would have been that shown by the dashed curve in Fig. 3(b). Obviously, this curve cannot be reconciled in any way with the observed spectrum.

We feel that the results presented show clearly that the frequency dependence of the lattice attenuation in CdS does undergo a transition from an $f^2$ to an $f^1$ dependence. That this transition signifies a change in regime intrinsic to the CdS crystal and not a structure-sensitive property is evidenced by the data accumulated so far for the low-frequency range (1–4 GHz). The results summarized in Fig. 2 for this range represent measurements on samples of different sources and different resistivities, all yielding much the same values for $\alpha_l$ and an identical Akhiezer $f^2$ dependence. It is unlikely that imperfections and/or impurities which are inactive in the attenuation process at low frequencies would become effective at higher frequencies, much less that such structural factors would *reduce* the attenuation below the extrapolated $f^2$ curve.

A detailed comparison of our $\alpha_l(f)$ data with theory[3,18] will be discussed elsewhere. At this point we shall mention only one conclusion. The transition between the Akhiezer and Landau-Rumer regimes is seen from Fig. 2 to occur around 4–5 GHz. This corresponds to a value of about 35 psec for the average lifetime $\tau$ of the thermal phonons responsible for the attenuation of the ultrasonic flux. Since there is no direct way of determining $\tau$, the common practice[5,9,19,20] in analyzing lattice attenuation data has been to estimate $\tau$ from the thermal conductivity, even though it is doubtful[19,21] whether the same thermal phonons are involved in the thermal-conductivity and ultrasonic-attenuation processes. The room-temperature value derived[22] from the thermal conductivity is 7 psec, considerably lower than the value of about 35 psec suggested by our $\alpha_l$ data. This indicates that the thermal phonons interacting with the ultrasonic flux are indeed different from those important in determining the thermal conductivity.

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