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To cite this article: D A Parshin 1993 *Phys. Scr.* **1993** 180

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# Soft Potential Model and Universal Properties of Glasses

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Received March 29, 1993; accepted March 31, 1993

## Abstract

It is shown that all the universal properties of glasses can be explained in the framework of the Soft Potential Model (SPM). At low temperatures (below a few kelvin) this model is equivalent to the well-known model of two-level systems of Anderson, Halperin, Varma and Phillips, which explains quite well the universal low-temperature properties of glasses. The SPM predicts that in addition to two-level systems there are soft harmonic oscillators in glasses which are responsible for their universal behavior, especially at higher temperatures.

Both the two-level systems and harmonic oscillators are described by anharmonic soft atomic potentials by the uniform way. They interact with phonons (or electric field) by means of bilinear terms with the same coupling constant for both types of low-energy excitations. The elastic interaction between soft atomic potentials leads to a density of the low-energy excitations, which does not depend on the concentration of defects in the glass structure.

## 1. Introduction

During the last 20 years it was established that different glasses exhibit universal properties, which are usually regarded as anomalous compared to those of the crystalline counterparts [1]. They include low-temperature specific heat, thermal conductivity, propagation of ultrasound, dielectric losses, electric and acoustic echo, heat release, and some other properties governed by low-energy modes.

The universal low-temperature properties of glasses (below a few kelvin) have been understood well in the framework of the model of Anderson, Halperin, Varma and Phillips (AHVP model) [2, 3]. This model postulates the existence of two-level systems (TLS's) in glasses with an almost constant density of states  $\bar{P}$ . There are several review articles where experimental data and their interpretation in the framework of the AHVP model are given [4–8].

However, above a few kelvin the universal properties of glasses deviate from the predictions of the AHVP model. The thermal conductivity shows a plateau around 10 K, which cannot be understood in terms of a constant density of tunneling states [9]. The sound velocity decreases linearly with temperature above a few kelvin [10]. In numerous experiments in different glasses and glasslike materials it has been observed the saturation of the heat release as a function of the onset temperature [11, 12]. Furthermore, there are an additional increase in the specific heat, and in the low frequency Raman scattering [13] indicating the existence of still another kind of low-frequency modes. Recent neutron measurements [14] have shown these to be soft harmonic

vibrations with a crossover to anharmonicity at the low-frequency end (at frequencies corresponding to several kelvin).

All these experimental facts indicate that there is a common basis for the low-temperature (below a few kelvin) and the higher-temperature (above a few kelvin) universal properties of glasses. And that the AHVP model describes only one part of it. In addition to the TLS's there are other low-energy excitations in glasses, which are also responsible for their universal properties at higher temperatures. In experiments of Grace and Anderson [15] and Brand and Löhneysen [16] it has been demonstrated that TLS's and these additional excitation have the same basic vibrational structure. The computer simulation of a glass of soft spheres shows clearly the existence of (quasi-) localized modes at the low-frequency end of the vibrational spectrum [17].

That is what just follows from the predictions of the soft potential model (SPM) which was proposed in [18] and developed further in [19–26]. The SPM explains all the universal low-temperature properties of glasses as well as the AHVP model [23]. But in this model besides the TLS's, soft harmonic oscillators (HO) exist too and both these types of low-energy excitations have a common basis, namely soft atomic potentials. The TLS's exist in the soft double-well potentials and the HO in the soft single-well ones. The soft harmonic excitations are just the low-energy excitations, which are responsible for some universal properties of glasses at higher temperatures [24, 25]. Both types of excitations are intimately related to each other. Due to the softness of the potentials internal strains easily transform one type to another.

Here we consider the universal properties of glasses and show that it is possible to describe all of them in the SPM.

## 2. Soft potential model

According to the soft potential model [18], the quasilocal low-frequency modes in glasses are described by the soft anharmonic oscillator potential

$$V(x) = \mathcal{E}_0 \left[ \eta \left( \frac{x}{a} \right)^2 + \xi \left( \frac{x}{a} \right)^3 + \left( \frac{x}{a} \right)^4 \right]. \quad (1)$$

Here  $x$  is the generalized coordinate of the soft mode having units of length,  $a$  is the characteristic length of the order of the interatomic spacing ( $a \simeq 1 \text{ \AA}$ ),  $\mathcal{E}_0$  is the binding energy of the order of  $\bar{M}v^2 \simeq 10 \text{ eV}$ ,  $\bar{M}$  being the average mass of atoms constituting the glass,  $v$  is the sound velocity. The values of the dimensionless parameters  $\eta$  and  $\xi$  are random

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due to fluctuations of the structural parameters of a glass. The soft potentials correspond to  $|\eta|, |\xi| \ll 1$ . The distribution function of these parameters for  $|\eta|, |\xi| \ll 1$  is given by [20]

$$P(\eta, \xi) = \frac{|\eta|}{2} \mathcal{P}_0, \quad (2)$$

where  $\mathcal{P}_0$  is a constant and the factor  $|\eta|$  describes so-called "sea-gull" singularity in the distribution of the parameter  $\eta$ . Due to elastic interaction between soft atomic potentials  $\mathcal{P}_0$  does not depend on the concentration of defects in the glass structure. It is determined by the normalized distribution function  $\Psi(\eta)$  of the parameter  $\eta$  when the interaction among defects is switched off. Its order of the value is given by  $\mathcal{P}_0 \simeq \Psi(0)/a^3$ , where  $\Psi(0) \simeq 0.1$  [23].

For  $|\xi|/\sqrt{\eta_L} < \eta_L/|\eta|$ , negative  $\eta$  and  $|\eta| > 3\eta_L$  (see Fig. 1) the two lowest levels in the potential (1) form a TLS with the energy distance  $E = \sqrt{\Delta_0^2 + \Delta^2}$ . The tunneling splitting  $\Delta_0$  and asymmetry  $\Delta$  are determined by:

$$\Delta_0 = W \exp \left[ -\frac{\sqrt{2}}{3} \left( \frac{|\eta|}{\eta_L} \right)^{3/2} \right], \quad \Delta = \frac{W}{\sqrt{2}} \frac{|\xi|}{\sqrt{\eta_L}} \left( \frac{|\eta|}{\eta_L} \right)^{3/2}. \quad (3)$$

Here  $\eta_L$  is an important small parameter of the model, and  $W$  is a characteristic energy in the potential (1) for  $\eta = \xi = 0$

$$\eta_L = (\hbar^2/2Ma^2\mathcal{E}_0)^{1/3} \approx 10^{-2}, \quad W = \mathcal{E}_0\eta_L^2 \approx k \cdot 10 \text{ K}. \quad (4)$$

Here  $M$  is an effective mass of the tunneling entity. The barrier height  $V$  between two minima in the double-well potential (1) for  $\Delta \ll V$  only depends on the value of  $|\eta|$

$$V = \frac{W}{4} \left( \frac{\eta}{\eta_L} \right)^2. \quad (5)$$

Just existence of the small parameter  $\eta_L$  in the model makes it possible to neglect all the higher order terms in the expansion (1). Therefore, all the soft atomic potentials in glasses can be described in this way. This remark means that the SPM is in fact a theory of quasilocal soft anharmonic excitations in glasses.

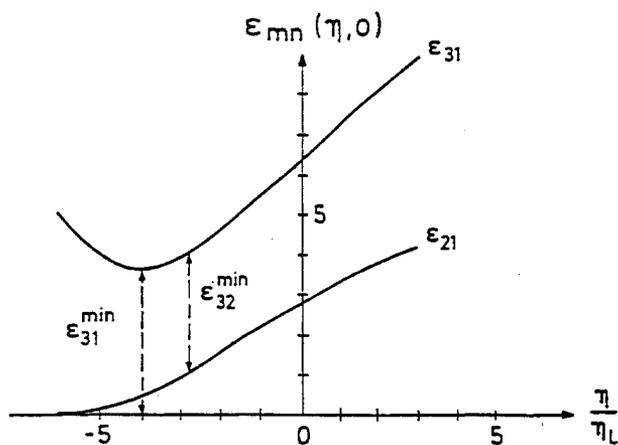


Fig. 1. The interlevel distance in the potential (1) (in the units of  $W$ ) as a function of  $\eta/\eta_L$  for  $\zeta = 0$ . The arrows indicate the minimal distance between the levels  $\varepsilon_{31}^{\min} = 3.65$ ,  $\varepsilon_{32}^{\min} = 3$ . These are the points of the absolute minimum of the functions  $\varepsilon_{31}(\eta, \zeta)$  and  $\varepsilon_{32}(\eta, \zeta)$  on the plane  $\eta, \zeta$ . The corresponding density of states makes a finite jump (from zero) at this energy – a so-called van Hove singularity [19].

The density of states of TLS's as a function of their energy  $E$  appears to be nearly a constant

$$F(E, p) = \left( \frac{2}{9} \right)^{1/3} \frac{\mathcal{P}_0 \eta_L^{5/2}}{W} \frac{1}{p \sqrt{1-p}} \ln^{-2/3} \frac{W}{E \sqrt{p}}, \quad (6)$$

$$p = \left( \frac{\Delta_0}{E} \right)^2.$$

It is only slightly (by the logarithmic factor) different from the corresponding distribution function in the AHVP model [5]. The order of its magnitude is determined by the value  $\mathcal{P}_0 \eta_L^{5/2}/W$ . The factor  $\eta_L^{5/2} \simeq 10^{-5}$ ,  $\mathcal{P}_0 \simeq 0.1/a^3$  and  $W/k$  is of the order of a few kelvin. As a result, the concentration of TLS's with energies below a few kelvin is of the order of  $10^{-6}$  from the concentration of atoms constituting the glass. That corresponds to the experimental data. The fundamental physical reason for the small parameter  $\eta_L$  is the small ratio of the electron mass  $m_e$  to the effective mass of the tunneling entity  $\eta_L \simeq (m_e/M)^{1/3}$ . In crystalline solids the parameter  $\eta_L$  determines a smallness of anharmonic processes. Thus, the small anharmonicity in crystalline solids and small concentration of TLS's in glasses have the same reason [23].

It is clear from Fig. 1, that the TLS's picture is lost for sufficiently small  $|\eta| \simeq \eta_L$  because the distance to the third level becomes comparable with the TLS energy  $E$ . This means that the TLS's alone cannot be responsible for excitations with energies larger than  $W$ . For these excitations in glasses single-well potentials are responsible too.

Besides the TLS's corresponding to double-well potentials with  $\eta < 0$ , eq. (1) describes also quasi-local anharmonic oscillators in single-well potentials with positive  $\eta$  and  $\eta > 9\xi^2/32$ . The excitations in the single-well potentials with  $\eta \gg \eta_L$  are nearly harmonic ones, the interlevel spacing being

$$E = 2W \sqrt{\frac{\eta}{\eta_L}}. \quad (7)$$

The density of states ( $n(E)$ ) of these excitations sharply increases with the increasing energy  $E$ . For  $E \gg W$  [20, 22]

$$n(E) = \frac{1}{6\sqrt{2}} \frac{\mathcal{P}_0 \eta_L^{5/2}}{W} \left( \frac{E}{W} \right)^4 \propto E^4. \quad (8)$$

The interaction of the soft atomic potentials (1) with a deformation  $\varepsilon$  is described by the bilinear term [23, 24]

$$V_{\text{int}}(x) = \mathcal{E}_0 \hat{H} \left( \frac{x}{a} \right) \varepsilon. \quad (9)$$

The dimensionless coefficient  $\hat{H} \simeq 1$ . This term describes both the interactions of TLS's and of HO with the strain field with the same coupling constant.

### 3. Low-temperature properties, $T \ll W/k$

In this section we consider the predictions of the SPM concerning the low-temperature properties of glasses. We show that with respect to these properties the SPM is equivalent to the AHVP model. Due to the existence of the absolute minimum distance between the second and the third levels in the potential (1) (see Fig. 1), all the low-temperature

properties of glasses for  $T \ll W/k$  in the SPM are determined by TLS's only. We begin with contribution of them to the specific heat.

### 3.1. Specific heat

Making use of distribution function (6) we obtain for TLS's contribution to the specific heat

$$C_{\text{TLS}}(T) = \pi^2 \left(\frac{2}{9}\right)^{1/3} \frac{\mathcal{P}_0 \eta_L^{5/2}}{W} k^2 T \ln^{1/3} \left( \frac{W}{kT} \sqrt{\frac{t_{\text{exp}}}{\tau_{\text{min}}(T)}} \right), \quad (10)$$

where  $\tau_{\text{min}}(T)$  is the relaxation time of the TLS [see eq. (12)] with  $\Delta = 0$  and  $E = 2kT$ . The specific heat has an almost linear temperature dependence (which coincide with the results of the AHVP model). It depends only weakly on the time of experiment:  $C(T) \sim \ln^{1/3}(t_{\text{exp}})$  [instead of  $C(T) \sim \ln(t_{\text{exp}})$  as in the AHVP model].

### 3.2. Ultrasonic properties

#### 3.2.1. TLS's contribution to the ultrasonic absorption.

There are two mechanisms of ultrasonic absorption in glasses: relaxation and resonant one. We begin with relaxation absorption. For  $\hbar\omega \ll kT$  it is due to the modulation of the TLS's energy  $E$  by the sound wave. It is determined as usual [4–6, 27]:

$$l_{\text{rel, TLS}}^{-1} = \frac{1}{\mathcal{V}} \sum_{\text{TLS}} \frac{D^2}{\rho v^3} \frac{1}{4kT \cosh^2(E/2kT)} \frac{\omega^2 \tau}{1 + \omega^2 \tau^2}, \quad (11)$$

where  $\mathcal{V}$  is the volume of the glass,  $\rho$  is the mass density,  $\omega$  is the ultrasound frequency and  $\tau$  is the TLS relaxation time (determined by one-phonon processes):

$$\frac{1}{\tau} = \frac{M^2 E^3}{2\pi \rho \hbar^4 v^5} \coth \frac{E}{2kT}. \quad (12)$$

The deformation potentials  $M$  and  $D$  of the TLS depend on the deformation potential  $\gamma$  which describes variation of the asymmetry  $\Delta$  under strain  $\varepsilon$

$$D = 2 \frac{\Delta}{E} \gamma, \quad M = \frac{\Delta_0}{E} \gamma, \quad \gamma = \frac{1}{2} \frac{\partial \Delta}{\partial \varepsilon}. \quad (13)$$

From eq. (9) we have the absolute value of  $\gamma$  [23, 24]

$$|\gamma| = \frac{1}{\sqrt{2}} \frac{|\dot{H}| W}{\eta_L^{3/2}} \sqrt{\frac{\eta}{\eta_L}} = \left(\frac{3}{4}\right)^{1/3} \frac{|\dot{H}| W}{\eta_L^{3/2}} \ln^{1/3} \frac{W}{E \sqrt{p}}. \quad (14)$$

As it follows from eq. (14)  $|\gamma|$  is of the order of  $|\dot{H}| W / \eta_L^{3/2}$  and logarithmically depends on the tunneling splitting  $\Delta_0$ . Therefore, in the tunneling region the deformation potential appears to be nearly a constant as in the AHVP model. For  $|\dot{H}| \simeq 1$ ,  $W/k \simeq 10$  K and  $\eta_L \simeq 10^{-2}$  we have  $|\gamma| \simeq 1$  eV. Thus, in the SPM  $|\gamma|$  has the usual value for glasses of the order of 1 eV [4–6].

Considering the relaxation ultrasound absorption (11) one usually discriminates between two limiting cases of  $\omega \tau_{\text{min}}(T) \ll 1$  and  $\omega \tau_{\text{min}}(T) \gg 1$ .

(\*)  $\omega \tau_{\text{min}}(T) \ll 1$ . In this case we have the usual result for the absorption coefficient to be independent of the temperature ("plateau region") [23]:

$$l_{\text{rel, TLS}}^{-1} = \frac{\pi C \omega}{2 v}, \quad C \equiv \frac{\mathcal{P}_0 \dot{H}^2 W}{\rho v^2 \sqrt{\eta_L}} \simeq \Psi(0) \eta_L^{3/2} \simeq 10^{-4}. \quad (15)$$

It depends linearly on  $\omega$  as in the AHVP model [4–6, 27].

(\*\*)  $\omega \tau_{\text{min}}(T) \gg 1$ . In this case we have [23]:

$$l_{\text{rel, TLS}}^{-1} = \frac{\pi^3 (36)^{1/3}}{96} \frac{\mathcal{P}_0 \dot{H}^4 W^3}{\rho^2 \hbar^4 v^8 \eta_L^{7/2}} \ln^{2/3} \left( \frac{W}{2kT} \right) (kT)^3. \quad (16)$$

The absorption is proportional to  $T^3 \ln^{2/3}(W/2kT)$  and independent of frequency. This result differs only slightly from the corresponding result of AHVP model [4–6, 27]. There is an additional logarithmical temperature dependence due to the factor  $\ln^{2/3}(W/2kT)$ .

For the resonant contribution to the sound absorption the TLS's with energy  $E = \hbar\omega$  are responsible [4–6]. Calculations give us the result [23]:

$$l_{\text{res, TLS}}^{-1} = \pi \frac{C \omega}{v} \tanh \frac{\hbar\omega}{2kT}. \quad (17)$$

The frequency and temperature dependencies of the absorption are identical with the results of the AHVP model [4–6].

3.2.2. TLS's contribution to the sound velocity. As is well known, TLS's also modify the sound velocity of glasses and determine its temperature dependence in the low-temperature region [4–6]. Relative contribution of TLS's to the sound velocity consists of two parts resonant and relaxation one. In the (\*\*) case the relaxation contribution is small enough. However, in the plateau region it is important and given by [23]

$$\left( \frac{v(T) - v(T_0)}{v(T_0)} \right)_{\text{rel, TLS}} = -\frac{3}{2} C \ln \frac{T}{T_0}, \quad (18)$$

In contrast, the resonant contribution is always important for both cases (\*) and (\*\*) [23]:

$$\left( \frac{v(T) - v(T_0)}{v(T_0)} \right)_{\text{res, TLS}} = C \ln \frac{T}{T_0}. \quad (19)$$

So, in the SPM we have the same logarithmical temperature dependence of the sound velocity as in the AHVP model [4–6].

We can see that for  $T \ll W/k$  the SPM reproduces all the results of the AHVP model and gives the limit of applicability of this model. Dimensionless parameter  $C$  determined in eq. (15) is equivalent to parameter  $\bar{P} \gamma^2 / \rho v^2$  in the AHVP model. Another parameter  $\mathcal{P}_0 \eta_L^{5/2} / W$  determines the TLS's density of states (6). In the SPM all the low-temperature properties of glasses are determined by these two parameters only (as in the AHVP model). The characteristic energy  $W$  cannot be obtained from low-temperature data, but it governs the higher-temperature universal properties. And now we go to consideration of the properties of glasses at higher temperatures:  $T \gg W/k$ .

## 4. Higher-temperature properties, $T \gg W/k$

For the higher-temperature universal properties of glasses the excitations with energies bigger than  $W$  are responsible. It can be both TLS's with  $E \gg W$  and HO with energies  $E \gg W$ .

### 4.1. Specific heat

At temperatures  $T \gg W/k$  the main contribution to the specific heat comes from the HO. The strongly rising density of states (8) leads to the specific heat of HO proportional

to  $T^5$  [22]

$$C_{\text{HO}}(T) = \frac{8\pi^6 k}{63\sqrt{2}} \mathcal{P}_0 \eta_L^{5/2} \left(\frac{kT}{W}\right)^5, \quad (20)$$

and therefore a minimum results in the temperature dependence of  $C(T)/T^3$  [20, 22]. Such a minimum is usually observed for a large number of glasses. From the position of this minimum  $T_{\text{min}}$  one can estimate the value of the characteristic energy  $W$ :  $W \approx (2 - 2.5)kT_{\text{min}}$ . For example, for  $\text{SiO}_2$ ,  $W/k \approx 4 - 5$  K.

#### 4.2. Ultrasonic properties

4.2.1. *Resonant absorption by HO.* For the resonant absorption of phonons with energies  $\hbar\omega \gg W$  the HO with energies  $E = \hbar\omega$  are responsible. The resonant absorption strongly depends on frequency  $\omega$  and does not depend on temperature [24]

$$I_{\text{res, HO}}^{-1} = \frac{\pi}{6\sqrt{2}} \frac{C\omega}{v} \left(\frac{\hbar\omega}{W}\right)^3 \propto \omega^4, \quad (21)$$

This result is also valid in the low-temperature region  $T \ll W/k$ .

4.2.2. *Relaxation absorption due to TLS's.* In this temperature region there are two sources of the relaxation absorption: due to TLS's with high barriers and due to HO. We begin with TLS's contribution to the relaxation absorption. The physical reason for the absorption is the same as in the case of low temperatures. And it is determined by the same expression [eq. (11)]. However, the relaxation processes are different. If at low-temperatures  $T \ll W/k$  the relaxation was due to tunneling through the barrier, at higher-temperatures  $T \gg W/k$  it is due to thermal activation processes:  $\tau = \tau_0 \exp(V/kT)$ . In the SPM there is a characteristic crossover temperature  $T_c$  between tunneling and activation processes where the rates of both processes coincide [21]:

$$kT_c \approx \left(\frac{3}{8}\right)^{4/3} W \ln^{1/3} \frac{1}{\omega\tau_0}. \quad (22)$$

For  $T > T_c$  the thermal activated hopping over the barrier responsible for the relaxation of the TLS and instead of eq. (15) we have [24]

$$I_{\text{rel, TLS}}^{-1} = \frac{\pi\omega C}{v} \left(\frac{kT}{W}\right)^{3/4} \ln^{-1/4} \frac{1}{\omega\tau_0}. \quad (23)$$

It increases with temperature  $\propto T^{3/4}$ .

4.2.3. *Relaxation absorption due to HO.* The physical reason for the absorption is the same as for TLS's: modulation of the interlevel distance of the HO. The absorption is given by a formula analogous to eq. (11) [25]

$$I_{\text{rel, HO}}^{-1} = \frac{1}{\mathcal{V}} \sum_{\text{HO}} \frac{D^2}{\rho v^3} \frac{1}{4kT \sinh^2(E/2kT)} \frac{\omega^2 \tau}{1 + \omega^2 \tau^2}. \quad (24)$$

Here  $\tau$  is the relaxation time of the HO and  $D$  is the deformation potential of HO describing variation of energy (7) with deformation  $\varepsilon$

$$D = \frac{\delta E}{\delta \varepsilon} = -12 \frac{\dot{H}W}{\eta_L^2} \zeta \left(\frac{W}{E}\right)^3. \quad (25)$$

The relaxation time  $\tau$  of the HO is determined by one-phonon processes, i.e., resonant absorption and emission of

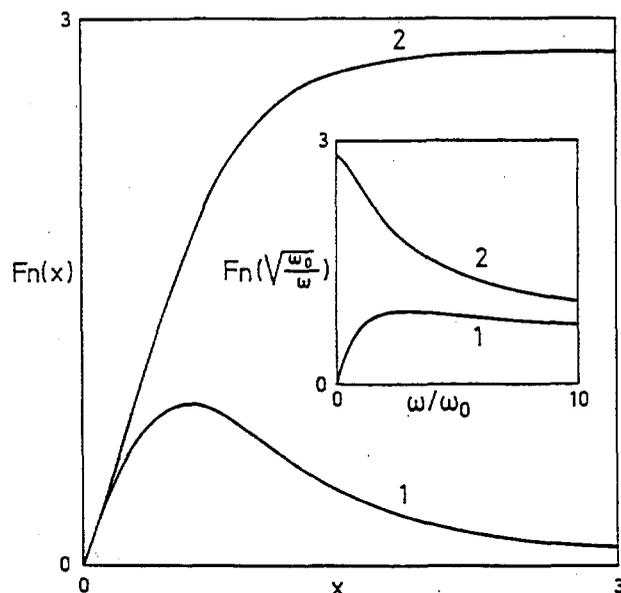


Fig. 2. The plot of the functions  $F_n(x)$  ( $n = 1, 2$ ) [eq. (28)]. Inset: the plot of the functions  $F_n(\sqrt{\omega_0/\omega})$  describing the frequency dependence of the ultrasound absorption and sound velocity due to HO in glasses,  $\hbar\omega_0 = W(E_0/E_c)^2$ .

phonons and does not depend on temperature:

$$\frac{1}{\tau} = \left(\frac{WH}{\eta_L^{3/2}}\right)^2 \frac{WE^2}{2\pi\rho\hbar^4 v^5} \equiv \frac{W}{\hbar} \left(\frac{E}{E_c}\right)^2, \quad (26)$$

$$E_c \equiv \frac{\sqrt{2\pi\rho\hbar^3 v^5}}{(|\dot{H}| W/\eta_L^{3/2})}.$$

For example, for  $\text{SiO}_2$ ,  $E_c/k \approx 50$  K.

As a result in the frequency region  $\hbar\omega \ll W(2kT/E_c)^2$  [25]

$$I_{\text{rel, HO}}^{-1} = \frac{16}{9} \frac{C\omega}{v} \frac{kT}{E_0} F_1\left(\frac{E_0}{E_c} \sqrt{\frac{W}{\hbar\omega}}\right), \quad (27)$$

where ( $n = 1, 2$ )

$$F_n(x) = x \left( \arccos \frac{x^2 - 1}{\sqrt{1 + x^4}} + (-)^n \ln \frac{x^2 + \sqrt{2x + 1}}{\sqrt{1 + x^4}} \right). \quad (28)$$

Here  $E_0 \approx 3W$  is a crossover energy between TLS and HO description of the spectrum in the potential (1) (see Fig. 1). The absorption increases linearly with temperature in the whole frequency region under consideration.\* The function  $F_1(x)$  is shown in Fig. 2. It is a non-monotonous function of its argument and has a maximum at  $x = x_m \approx 0.6$  [ $F_1(x_m) \approx 0.89$ ]. As a result the internal friction  $Q^{-1} = v/\omega l$  is a non-monotonous function of the frequency  $\omega$ . It has a maximum in the Brillouin scattering region at  $\hbar\omega_m \approx 2.8W(E_0/E_c)^2$ . At low frequencies  $\hbar\omega \ll W(E_0/E_c)^2$ , the absorption  $I_{\text{rel, HO}}^{-1} \propto \omega^2$  and small enough in comparison with contribution from TLS's [eq. (23)]. However, at higher frequencies (in the Brillouin scattering region) it can be comparable with TLS's contribution.

At still higher frequencies  $\hbar\omega \gg W(2kT/E_c)^2$  the absorption is proportional to the temperature squared and does

\* However, with increasing temperature two-phonon processes in relaxation of HO become important and absorption saturates and becomes independent of temperature.

not depend on the frequency.

$$l_{\text{rel, HO}}^{-1} = \frac{32\pi^2 C\omega}{27} \underbrace{\frac{W}{v} \left(\frac{kT}{E_c}\right)^2}_{\ll 1} \propto T^2. \quad (29)$$

**4.2.4. Contribution of TLS's to the sound velocity.** In the thermal activated region  $T > T_c$  the contribution of TLS's to the sound velocity is due to thermal activation processes and different from tunneling regime (18) [25]

$$\left(\frac{\delta v}{v}\right)_{\text{rel, TLS}} = -\frac{4}{3}C \left(\frac{kT}{W}\right)^{3/4} \ln^{3/4} \frac{1}{\omega\tau_0}. \quad (30)$$

It decreases nearly linearly with temperature ( $\propto T^{3/4}$ ) and logarithmically depends on frequency.

**4.2.5. Contribution of HO to the sound velocity.** In the classical region  $\hbar\omega \ll kT$  the contribution of HO to the sound velocity of glasses is due to modulation of the inter-level spacing (7) by the sound wave. It can be presented as a sum of two terms [25]

$$\left(\frac{\delta v}{v}\right)_{\text{HO}} = \frac{1}{2\rho v^2} \frac{1}{\mathcal{V}} \sum_{\text{HO}} \left( \frac{1}{2} \frac{\delta^2 E}{\delta \varepsilon^2} \coth \frac{E}{2kT} - \frac{D^2}{4kT \sinh^2(E/2kT)} \frac{1}{1 + \omega^2 \tau^2} \right). \quad (31)$$

The first (non-inertial) contribution is independent of the HO relaxation time  $\tau$  and on the frequency  $\omega$ . It is analogous to the resonant contribution of the TLS's to the sound velocity. The second (inertial) contribution depends on the HO relaxation time  $\tau$ . It is analogous to the relaxation contribution of the TLS's.

Non-inertial contribution is determined by HO with energies  $E \simeq E_0$ :

$$\left(\frac{v(T) - v(T_0)}{v(T_0)}\right)_{1, \text{HO}} = -\frac{4\sqrt{2}}{3} \frac{Ck}{E_0} (T - T_0). \quad (32)$$

For the inertial contribution in the frequency range  $\hbar\omega \ll W(2kT/E_c)^2$  we have [25]:

$$\left(\frac{\delta v}{v}\right)_{2, \text{HO}} = -\frac{8}{9} \frac{Ck}{E_0} TF_2 \left( \frac{E_0}{E_c} \sqrt{\frac{W}{\hbar\omega}} \right). \quad (33)$$

Both contributions are negative and linearly depend on temperature. The inertial contribution decreases monotonously with frequency and maximal at low frequencies  $\hbar\omega \ll W(E_0/E_c)^2$ . In this region the total contribution of HO is

$$\left(\frac{v(T) - v(T_0)}{v(T_0)}\right)_{\text{tot, HO}} = -\frac{28\sqrt{2}}{9} \frac{Ck}{E_0} (T - T_0). \quad (34)$$

## 5. Comparison with experiment

From the consideration presented above we can see that in the SPM all the universal properties of a particular glass are determined by three parameters only. The first one is  $\mathcal{P}_0 \eta_L^{3/2}/W$  characterizing density of states of soft excitations (both TLS's and HO). The second one is  $\dot{H}W/\eta_L^{3/2}$  determining their deformation potential. And the third one is the

characteristic energy  $W$ . First two of them can be determined from low-temperature experiments as usual. The third one governs higher temperature properties. It can be estimated, for example, from position of the minimum in the specific heat  $C(T)/T^3$ . After that all three parameters become fixed and to explain other higher temperature properties we have no fitting parameters any more. Nevertheless, this procedure gives startling results. It describes with a good accuracy the position of the plateau in the thermal conductivity of vitreous silica and amorphous selenium [24]. It reproduces quite well temperature dependence of ultrasonic and hypersonic attenuation and frequency dependence of the mean free path of high frequency phonons in  $a - \text{SiO}_2$  and their absolute values [24].

The theory gives also the correct value for the slope in the linear temperature dependence of the sound velocity in chalcogenide glasses  $a - \text{As}_2\text{S}_3$ ,  $a - \text{As}_2\text{Se}_3$  and  $a - \text{Se}$  [28, 29]. In these glasses there are some reasons to believe that the linear temperature dependence of the sound velocity is due to HO contribution (34) only [25].

The SPM by the natural way explains also the experimental data on low-frequency inelastic light scattering in glasses. Quasilocal soft modes modulate the local dielectric susceptibility of the glass  $\delta\varepsilon_{ik} \propto \alpha_{ik} x$  and therefore, causes the inelastic light scattering. As a consequence of the fluctuation-dissipation theorem the frequency and temperature dependence of the Stokes Raman scattering intensity are determined by  $I(\omega, T) \propto [1 + n(\omega)] l^{-1}(\omega, T)/\omega$ . Here  $n(\omega)$  is the Bose distribution function  $n(\omega) = [\exp(\hbar\omega/kT) - 1]^{-1}$  for temperature  $T$  and  $l(\omega, T)$  is the mean free path of phonons with frequency  $\omega$  at temperature  $T$ . The inverse mean free path  $l^{-1}$  for  $T \gg W/k$  are determined by the sum of expressions (21), (23) and (27). And for reduced Raman scattering intensity  $l\omega^{-1}[n(\omega) + 1]^{-1} \propto l^{-1}(\omega, T)/\omega^2$  we have a minimum in the frequency dependence. Position of this minimum is determined by value of  $W$  and slightly depends on temperature. For example, in  $a - \text{SiO}_2$  where one can neglect contribution of HO [eq. (27)] at these frequencies we obtain  $\hbar\omega_{\text{min}} \approx 1.6W(kT/W)^{1/4}$ . For  $T = 80 \text{ K}$  and  $W/k = 4 \text{ K}$  the minimum situated at  $\omega/2\pi \approx 10 \text{ cm}^{-1}$  what coincides with experiment [13]. This approach gives also the same depolarization ratio in the whole frequency region because tensor  $\alpha_{ik}$  is assumed to be nearly the same for all soft modes both TLS's and HO [compare with eq. (9)].

The SPM explains also the heat release experiments in glasses [26]. At low onset temperatures  $T_1 < T_c$  it leads to results that are only slightly different from predictions of the AHVP model. But they are in better agreement with experiment. At higher temperatures  $T_1 > T_c$  the SPM predicts the saturation of the heat release as a function of the onset temperature. The saturation has been observed in numerous experiments in glasses and glasslike materials [11, 12]. The reason is fast thermal activation processes at temperatures  $T > T_c$ . It is noteworthy that measurements of the heat release in glasses give the unique possibility to determine not only the characteristic energy  $W$  but its whole distribution function. The distribution function has usually several peaks corresponding different types of TLS's in the glass. The position of the main maximum in the distribution coincides quite well with the value of  $W$  obtained from specific heat data.

In the SPM it is possible to relate the TLS's density of states  $\bar{P}$  and the deformation potential  $\gamma$  with the macroscopic parameters of the glass: the sound velocity  $v$ , the mass density  $\rho$  and the average mass of the atoms  $\bar{M}$ . If these macroscopic parameters are changed under pressure or due to changing of the chemical composition of the glass, or in some other ways, the corresponding variation of the TLS's parameters coincides well with the prediction of the theory [23].

Strongly rising resonant scattering of phonons due to HO [eq. (21)] lead to that at some energy  $E_d$  the mean free path of phonons becomes equal to their wavelength [20, 30]. The value of this energy is determined by expression [30]

$$E_d \approx (0.6 - 0.75)WC^{-1/3}. \quad (35)$$

For example, for  $a - \text{SiO}_2$ ,  $E_d/k \approx 45$  K. At higher energies excitation from one oscillator can jump to other oscillators directly on the distance of the wavelength (compare with Einstein model [31]). The picture of independent quasilocal harmonic excitation in this case is lost. Phonons and HO with energies  $E > E_d$  cannot be considered independently any longer. They become intermixed with each other. Above this energy the total density of states should be reconstructed. One can believe that just this phenomenon responsible for the bump in the specific heat  $C(T)/T^3$  at temperature of the order  $E_d/5k$ , for the rise of the thermal conductivity above the plateau and for the "boson" peak at frequency  $\hbar\omega \simeq E_d$  in the Raman scattering in glasses.

## 6. Conclusion

In the framework of the SPM it is possible by the uniform way to explain all the universal properties of glasses; many of them remained unexplained during the last 15 years. It brings in evidence that soft atomic potentials really exist in the glass structure and they are responsible for the universal properties of glasses. The SPM being in fact the theory of soft atomic potentials gives the adequate description of these properties. It is a powerful tool for understanding different phenomena in glasses and its application very often brings about splendid results.

## Acknowledgements

Useful discussions with G. Bellessa, J. F. Berret, U. Buchenau, Yu. M. Galperin, V. L. Gurevich, S. Hunklinger, V. G. Karpov, G. Kasper, H. von Löhneysen, V. I. Mel'nikov, J. Pelous, S. Sahling, H. R. Schober, J. P. Sethna, R. Vacher and G. Weiss are gratefully acknowledged. I thank the

CNRS (France) for financial support and the Laboratoire du Science des Matériaux Vitreux, Université Montpellier II for hospitality during my stay in France.

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