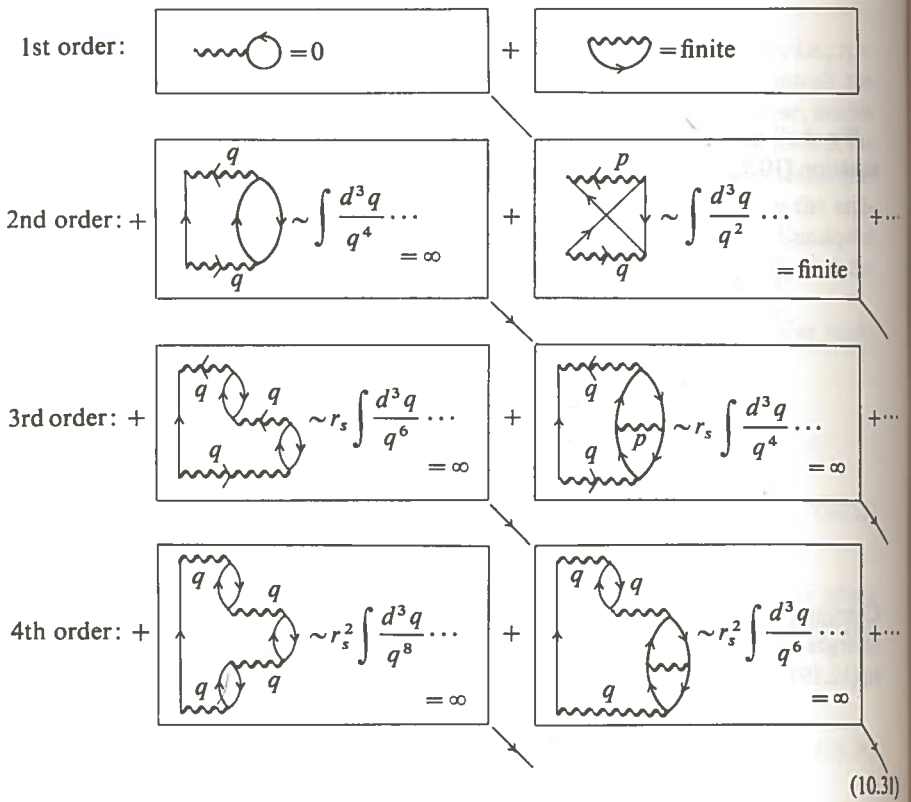
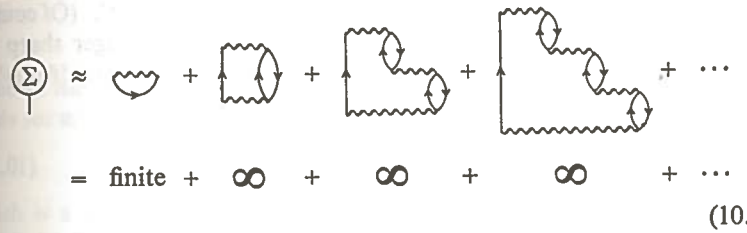
A look at the higher orders shows the same behaviour: most of the diagrams diverge. This appears to be a slightly discouraging start. However, the situation is saved by the partial summation method as follows: First of all, we observe that each term in the self-energy is proportional to some power of r_s ($\sim 1/k_F$—see §10.3). To see this, consider the second-order self-energy term in (9.66). To make the integral dimensionless, express quantities in units of k_F, i.e., set $q=q'k_F$, $\epsilon_l=\epsilon_l' k_F^2$, etc. Then we find that (9.66) $\sim k_F^3 \times k_F^3 \times k_F^{-4} \times k_F^{-2}=1$, so that (9.66) has no explicit dependence on k_F. Now look at a third-order term—e.g., the fifth or sixth diagram on the right of (10.8). After doing the frequency integrals, we find, compared with the second-order term, an extra V_q ($\sim k_F^{-2}$), an extra energy denominator ($\sim k_F^{-2}$) and an extra integral over k ($\sim k_F^3$), so that we have an extra factor $k_F^{-1} \sim r_s$. Thus all third-order terms are $\sim r_s$. In general, any nth-order term $\sim r_s^{n-2}$.

Next, we arrange the diagrams according to degree of divergence (=number of factors q^2 in the denominator of the integrand) and dependence on r_s:



(10.31)

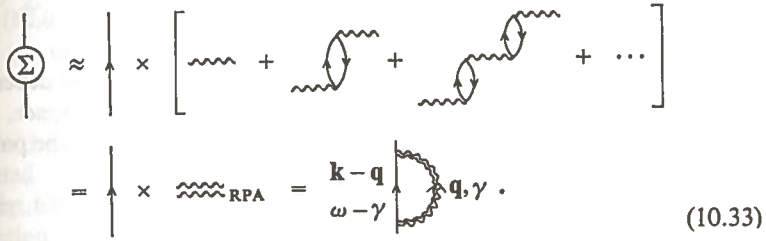
The first-order bubble has $\mathbf{q}=0$, so it vanishes by (10.28); that is, the bubble is cancelled by the positive background. We see that the digarams can be arranged so that in each order of divergence they form a power series in r_s (see along diagonal lines). Thus, for small r_s, i.e., high-density limit, the dominant infinite terms are just those of lowest order in r_s. Hence in the high density limit, the self-energy series is just the sum



(10.32)

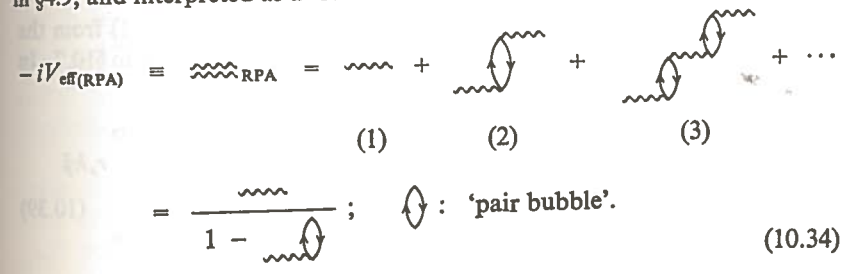
i.e., the sum over all diagrams of the repeated pair-bubble or 'ring' type. The remarkable thing is that this sum over an infinite number of infinite terms can actually be carried out, and it gives a finite result! Approximation (10.32) for the self-energy is called the 'random phase approximation' or 'RPA', for historical reasons.

The sum over rings is easy. Factoring out a free propagator from each diagram in (10.32) (this is not quite straightforward for the oyster, since it is a special case, by the rules of Table 9.1; however, it is legitimate) gives



(10.33)

The double wiggle is the 'effective interaction' in RPA, which was mentioned in §4.9, and interpreted as a 'screened' interaction between two particles:



(10.34)