Physics 216: Topics in many-body theory, Spring 2016

Problem set 4: assigned 3/31/16, due 4/14/16 (either in class or by 5 pm to 523 Birge)

1. Read Auerbach appendix B or another text on response functions and prove what is sometimes called the "fluctuation-dissipation theorem" relating the response function $R(q,\omega)$ (also known as the retarded correlation function) to the spin structure factor $S(q,\omega)$:

$$S(q,\omega) = -\frac{2}{1-e^{-\omega/kT}}\mathrm{Im}R(q,\omega). \tag{1}$$

Recall the definitions of these two quantities: S is the correlation function $\langle \mathbf{s}(x_1,t_1)\mathbf{s}(x_2,t_2)\rangle$, while R is defined in class as the response of an observable to a small perturbation in the Hamiltonian. Note that the sign convention used in Auerbach differs by a $-$ from what you may find elsewhere.

2. Disorder: I said in class that the model in 1D with random nearest-neighbor hopping and no onsite potentials is a little special: unlike the model that Anderson studied with random on-site potentials, the purely hopping model

$$H = \sum_{i=1}^{N} t_i (c_i^\dagger c_{i+1} + \text{h.c.}) \tag{2}$$

has a delocalized critical state at $E=0$. Work with closed boundary conditions for simplicity and N **even** so that the chain is bipartite.

(a) Show that this model has an exact "particle-hole" symmetry: if there is an eigenstate with energy E, then there is also an eigenstate with energy $-E$.

(b) Write the Hamiltonian matrix for the case $N=4$ and show that its determinant is a perfect square. It is generally nonzero, but it turns out that there is a singular peak in the density of states of a large system near $E=0$, and we can try to understand this as follows:

(c) Taking the continuum limit of this problem at low energy (see Balents and Fisher, Physical Review B 56, 12970 (1997), if you wish to know the details) leads to a 1D Dirac-like equation with random mass:

$$H_c = \int dx\, \psi^\dagger h(x) \psi \tag{3}$$

with

$$h = -i\sigma^z \partial_x + m(x)\sigma^y. \tag{4}$$

Find a two-component wavefunction that is a zero-energy eigenstate of this Hamiltonian. Hint: the one-component (i.e., scalar wavefunction)

$$\phi(x) = e^{\int^x dx\, m(x)} \tag{5}$$

has the nice property

$$\partial_x \phi(x) = m(x)\phi(x), \tag{6}$$

so use this and some appropriate sign changes to make a two-component wavefunction. The interesting case is when the random mass has mean zero. Argue that in this case your wavefunction is

not exponentially localized but power-law, at least for a well-behaved distribution. (Don't worry about normalizing your wavefunction.)

3. This should not need long calculations, just some thinking. For a classical 2D electron moving in a magnetic field, find the "guiding-center coordinates" (the x and y coordinates of the center of a cyclotron operator, which are constants of the motion). As quantum operators, do these commute with the Hamiltonian (choose rotational gauge for the magnetic field)? Do they commute with each other? What happens in Landau gauge, where the vector potential is of the form $(0, Bx, 0)$?

4. In class/recitation I said that "spin is not conserved in solids". We would like to make a very rough estimate, assuming that total angular momentum is conserved, of how long it takes spin and orbital angular momenta to interchange. Use quantum mechanics to estimate the magnitude of the $\mathbf{L} \cdot \mathbf{S}$ term for $2p$ electrons in an atom of atomic number Z (you can ignore other spin-orbit terms). Then assume that at $t = 0$ an orbital "pump" of angular momentum prepares an electron in the maximum L_z state within the $2p$ multiplet and with spin state $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$. Note that this state is not an energy eigenstate. Project this state onto spin-orbit eigenstates. What is the period of the resulting oscillations in L_z driven by spin-orbit coupling?